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AIRCRAFT ARMAMENT FOR AIR-TO-GROUND OPERATIONS (S)

PROJECT VISTA

CALIFORNIA INSTITUTE OF TECHNOLOGY

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Pasadena, California

November 30, 1951

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FOREWORD

This report was prepared by R. M. Stevens of the Cornell Aeronautical Laboratories during his association with Project Vista during the summer and fall of 1951. It represents the opinions of the author and may not in detail reflect the viewpoints of Project Vista.

B. H. Sage

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AIRCRAFT ARMAMENT FOR AIR-TO-GROUND OPERATIONS

INTRODUCTION

Of late, extensive and valuable studies have been made of the aircraft weapons systems in their undisputed, but not necessarily most productive role, air-to-air combat. Comparatively little attention has been given to their other domains of usefulness, in particular, that of tactical air-to-ground operations.

A comprehensive and detailed study of this operation is beyond the scope of this project. However, it is believed necessary to identify sufficiently well the governing physical phenomena that substantial bases are provided for recommendations either for further studies or specific actions.

In particular, this report will be concerned with aircraft armament, the ammunition and propellant system carried aboard the airplane. However, the other parts of the system, the target, the tactical situation, the airplane, the fire control system, the navigation system, and the general logistics of the operation will be considered wherever the characteristics of the armament cannot be considered independently. Specifically, this report will

1. Submit a measure of aircraft armament effectiveness, I_k
2. Examine systems and tactical parameters of which I_k is a function, and indicate methods of maximizing I_k
3. Compare I_k's of various aircraft armament systems
4. Recommend specific actions toward obtaining the maximum aircraft armament effectiveness

A Measure of Aircraft Armament Effectiveness

It is submitted without argument that an adequate measure of aircraft armament effectiveness will be the ratio of the number of kills obtained against specified targets to the weight of the armament installation which must be carried by the aircraft to obtain those kills. Symbolically

$$I_k = \frac{|N_k|_T}{W_o} = \frac{N_R(|P_k|_T)}{W_o} = N_R \frac{(|P_{(k/h)T}|)}{w_h} |P_h|_T \frac{w_h}{W_o} = |P_h|_T \frac{|P_{(k/h)}|_T}{w_h} \frac{W_h}{W_o}$$

where:

W_o = Weight of armament systems installation

w_h = Weight of warhead per round

W_h = Total weight of warhead in W_o

N_R = Number of rounds of ammunition in W_o

$|N_k|_T$ = Number of kills obtained against target, T, in W_o

$|P_k|_T$ = Probability of kill per round against target, T

$P_{(k/h)T}$ = Probability of kill per hit against target, T

$|P_h|_T$ = Single-shot probability of hit per round against target, T

The effectiveness index equals the product of the probability of hit per round, the probability of kill per hit per pound of warhead, and the ratio of the total weight of warhead to the total weight of armament system installation.

In other EngOrd reports, the inverse of the product of hit probability and the ratio of total warhead weight to ordnance system weight has been called the ordnance logistic factor, L. We shall adopt the same terminology, but since in this case we are considering in W_o only part of the ordnance system weight (ignoring the airplane weight) we shall say that:

$$I_k = \frac{1}{L_w} \frac{|P_{(k/h)}|_T}{w_h}$$

where:

L_w = The aircraft armament logistic factor

For the relationship between L and L_w see (9).

Examination of Parameters

$\frac{P_{(k/h)T}}{w_h}$ - Probability of kill per hit per unit warhead weight.

The parameter $\frac{P_{(k/h)T}}{w_h}$ is principally a function of the type of target, the type of warhead, the striking velocity, the striking attitude, and the type of fuzing. For simplicity, it will be assumed that comparatively this parameter is invariant among all aircraft armament propellant systems capable of delivering equal warhead weight, w_h.

L_w - Aircraft armament logistic factor.

L_w is a function of two parameters, which in turn are functions of many variables peculiar to the operation, as follows:

$|P_h|_T$ = Probability of hit per round against target, T, is a function of:

R = Slant range from point of release to target

$T = T (A_T, x_T, y_T)$ = Target (area, shape, normal to trajectory)

σ_i = Dispersions of round due to all causes other than release error along the sight line

σ_g = Dispersion of round due to release errors along the sight line, in turn a function of:

R = Slant range from point of release to target

V_a = Airplane velocity at release

V_r = Velocity of round

θ = Airplane dive angle at release

$\Delta\mathcal{E}_{si}$ = Release errors along sight line, in range, dive angle, and airplane speed

W_h/W_o

The ratio of total warhead weight to the ordnance system weight is a function of the ratios of round, gun, installation and control weights to the weight of the warhead per round $\left(\frac{W_R}{W_h}, \frac{W_g}{W_h}, \frac{W_i}{W_h}, \frac{W_a}{W_h}\right)$ which in turn are functions of variables peculiar to the operation as follows:

$\frac{W_R}{W_h}$ = Ratio of round weight to warhead weight, a function of:

V_r = Velocity of round

D_h = Caliber of round

R_t = Type of round

$\frac{W_g}{W_h}$ = Ratio of gun (or propelling system) weight to warhead weight, a function of:

V_r = Velocity of round

D_h = Caliber of round

N_g = Number of guns

N_R = Number of rounds

r_g = Rate of fire

G_i = Type of guns

$\frac{W_i}{W_h}$ = Ratio of installation weight to warhead weight a function of:

W_g = Weight of gun

D_h = Caliber of round

N_g = Number of guns

N_R = Number of rounds

r_g = Rate of fire

G_i = Type of guns

I_i = Type of Installation

$\frac{W_c}{W_h}$ = Ratio of control weight to warhead weight, a function of rates of change of the fundamental variables as well as of the variables.

There are other fundamental variables and relationships than those listed above. However, it is believed that enough have been recognized for fairly accurate general comparison of aircraft armament systems, yet the number has been kept sufficiently low that fairly simple analytic expressions may be derived.

Thus we say:

$$P_h = f\left(R, T, \sigma_i, \sigma_g\left(R, V_a, V_r, \theta, \Delta\varepsilon_{si}\right)\right)$$

$$W_h/W_o = g\left(\frac{W_R}{W_h}(V_r, D_h, R_i), \frac{W_g}{W_R}(V_r, D_h, N_r, \tau_g, G_i), \frac{W_i}{W_h}(W_g, W_h, I_i)\frac{W_z}{W_h}\right)$$

$$L_w = 1/gf$$

$$I_k = K/gf$$

It will be observed that f and g are functions of common variables. Therefore, P_h and W_h/W_o cannot be treated independently in maximizing L_w.

An Expression for P_h

Assume a rectangular target of width, w, and length, l, (normal to the target). Assume that the dispersion of the weapons system may be represented by a linear standard deviation of σ_{yR} in y (width) and σ_{xR} in x (length), where σ_x and σ_y are measured in mils.

Then:

$$P_h = P_{hx} P_{hy} = \frac{1}{\sqrt{2\pi}\,\sigma_x R}\int_{-l/2}^{l/2} e^{-1/2\left(\frac{x}{\sigma_x R}\right)^2} dx \cdot \frac{1}{\sqrt{2\pi}\,\sigma_y R}\int_{-w/2}^{w/2} e^{-1/2\left(\frac{y}{\sigma_y R}\right)^2} dy$$

$$= \frac{1}{2\pi\sigma_x\sigma_y R^2}\int_{-l/2}^{l/2} e^{-1/2\left(\frac{x}{\sigma_x R}\right)^2} dx \int_{-w/2}^{w/2} e^{-1/2\left(\frac{y}{\sigma_y R}\right)^2} dy$$

For values of $\frac{h}{2\sigma R}$ up to 1.0, the following relation

$$\int_{-h/2}^{h/2} e^{-1/2\left(\frac{z}{\sigma_z R}\right)^2} dz = \sigma_z R \int_{-h/2\sigma_z R}^{h/2\sigma_z R} e^{-1/2\, y^2} dy \cong h$$

does not introduce errors greater than 15%, the error decreasing with decreasing values of $h/2\sigma_z R$.

Thus for $\frac{h}{2\sigma_z R} \ll 1.0$:

$$P_h \cong \frac{2}{\pi}\,\frac{l}{2\sigma_x R}\,\frac{w}{2\sigma_y R}$$

with errors not greater than 35%.

If it is assumed that the number of rounds required per hit is equal to the reciprocal of the single-shot hit probability, to the nearest higher integer,

$$N_R = \frac{1}{P_h}$$

for values of $\frac{h}{2\sigma_z R} < 1.0$, the maximum error will not be greater than 50%, and this error will be greatest for the least number of rounds required (1.5 required by approximation as against 3 required by exact expression for $\frac{l}{2\sigma_x R} = \frac{w}{2\sigma_y R} = 0.99$).

Inasmuch as for the targets and ranges with which we are most concerned, $\frac{h}{2\sigma_z R} \ll 1.0$, it is considered that the following expression for hit probability is a satisfactory approximation.

$$P_h \doteq \frac{2}{\pi}\,\frac{l}{2\sigma_x R}\,\frac{w}{2\sigma_y R} = \frac{A_T}{2\pi R^2}\cdot\frac{1}{\sigma_x \sigma_y} \qquad \left[\begin{array}{l}\frac{l}{2\sigma_x R} < 1 \\ \frac{w}{2\sigma_y R} < 1\end{array}\right]$$

In the above expression, the variables which are not fundamental are σ_x and σ_y. σ_x is a function of both σ_i and σ_g, while σ_y is primarily a function only of σ_i as defined above. It will be assumed that for each type of aircraft armament σ_i is a constant, the value averaged from firing tests. However, as defined above,

$$\sigma_g = \sigma_g(R, V_a, V_R, \theta, \Delta\varepsilon_{ai})$$

so that $\sigma_y = \sigma_y(\sigma_i) = const = \sigma_i$

$$\sigma_x = \sigma_x(\sigma_i, \sigma_g) = \sigma_x(\sigma_i, \sigma_g(R, V_a, \theta, V_R, \Delta\varepsilon_{ai}))$$
$$= \sqrt{\sigma_i^2 + \sigma_g^2}$$

With the above assumption:

$$P_h \doteq \frac{A_T}{2\pi R^2}\cdot\frac{1}{\sigma_i\sqrt{\sigma_i^2 + \sigma_g^2}}$$

σ_y

It may be shown that for a vacuum trajectory (see Figure 1) the trajectory drop may be expressed as:

$$\mathcal{E} = \frac{m}{R} = \frac{g}{2}\,\frac{R}{(V_{av})^2}\cos\theta$$

where:

$\mathcal{E}$ = Trajectory drop (ft./ft.)

m = Trajectory drop (ft.)

R = Slant range (ft.)

V_{av} = Average velocity of projectile (ft./sec.) over slant range, R.

θ = Angle of airplane flight path to horizontal at time of firing.

V_a = Airplane velocity at time of firing.

And

$$\frac{\partial \mathcal{E}}{\partial R} = \frac{g}{2}\,\frac{R}{V_{av}^2}\cos\theta\left[\frac{1}{R} - \frac{2}{V_{av}}\,\frac{\partial V_{av}}{\partial R}\right]$$

$$\frac{\partial \mathcal{E}}{\partial V_a} = -g\cdot\frac{R}{V_{av}^3}\cos\theta\,\frac{\partial V_{av}}{\partial V_a}$$

$$\frac{\partial \mathcal{E}}{\partial \theta} = -\frac{g}{2}\,\frac{R}{V_{av}^2}\cos\theta\left[\tan\theta + \frac{2}{V_{av}}\,\frac{\partial V_{av}}{\partial \theta}\right]$$

(See Figure 2)

If it is assumed that $\frac{\partial \mathcal{E}_i}{\partial \rho} = \frac{\Delta \mathcal{E}_i}{\Delta \rho} = \frac{\sigma_i}{\Delta \rho}$

$$\sigma_R = \Delta \mathcal{E}_R = \frac{\Delta R}{R}\,\frac{g}{2}\,\frac{R\cos\theta}{V_{av}^2}\left[1 - \frac{2R}{V_{av}}\,\frac{\partial V_{av}}{\partial R}\right]$$

$$\sigma_{V_a} = \Delta \mathcal{E}_V \doteq \frac{-2\Delta V_a}{V_{av}}\,\frac{g}{2}\,\frac{R\cos\theta}{V_{av}^2}\,\frac{\partial V_{av}}{\partial V_a} = -2\frac{\Delta V_a}{V_a}\,\frac{V_a}{V_{av}}\,\frac{g}{2}\,\frac{R\cos\theta}{V_{av}^2}\,\frac{\partial V_{av}}{\partial V_a}$$

$$\sigma_\theta = \Delta \mathcal{E}_\theta \doteq -\frac{\Delta\theta}{\theta}\,\frac{g}{2}\,\frac{R\cos\theta}{V_{av}^2}\left[\theta\tan\theta + \frac{2\theta}{V_{av}}\,\frac{\partial V_{av}}{\partial \theta}\right]$$

and that these release errors are the principal ones not included in σ_i,

then: $\sigma_y \doteq \sqrt{\sigma_R^2 + \sigma_V^2 + \sigma_\theta^2}$

$$\doteq \frac{g}{2}\frac{R\cos\theta}{V_{av}^2}\sqrt{\left[\frac{\Delta R}{R}\left(1 - \frac{2R}{V_{av}}\frac{\partial V_{av}}{\partial R}\right)\right]^2 + \left[\frac{2\Delta V_a}{V_a}\frac{V_a}{V_{av}}\frac{\partial V_{av}}{\partial V_a}\right]^2 + \left[\frac{\Delta\theta}{\theta}\left(\theta\tan\theta + \frac{2\theta}{V_{av}}\frac{\partial V_{av}}{\partial\theta}\right)\right]^2}$$

Assume for a moment that V_{av} may be chosen arbitrarily, independent of R, V_a, and θ, then:

$$\sigma_g = \frac{g}{2}\frac{R\cos\theta}{V_{av}^2}\sqrt{\left(\frac{\Delta R}{R}\right)^2 + (\Delta\theta\tan\theta)^2}$$

and
$$P_h = \frac{A_T V_{av}^2}{\pi g R^3\cos\theta}\cdot\frac{1}{\sigma_i\sqrt{\left(\frac{2V_{av}^2\sigma_i}{gR\cos\theta}\right)^2 + \left(\frac{\Delta R}{R}\right)^2 + (\Delta\theta\tan\theta)^2}}$$

Here we observe that if σ_i is small relative to σ_g, (say $\sigma_i \geqslant 3\sigma_g$),

and $\frac{h}{2\sigma_z h} < 1$, P_h increases linearly with A_t

increases as the square of V_{av}

increases as the reciprocal of the cube of the range

increases as the reciprocals of σ_i and $\sqrt{\Delta\mathcal{E}_{si}^2}$

increases as the reciprocal of $\cos\theta$

If, on the other hand σ_i is large relative to σ_g (say $\sigma_i \geqslant 3\sigma_g$),

and $\frac{y}{2\sigma_2 R} < 1$, P_h increases linearly with A_t

increases as the reciprocal of the square of the range

increases as the reciprocals of σ_i and $\sqrt{\Delta\mathcal{E}_{si}^2}$

Now, $\frac{W_h}{W_o}$ is also a function of V_{av} so without further exploration of their relationship, the variation of I_k with V_a cannot be stated. However, $\frac{W_h}{W_o}$ is not a function of the other variables except as V_{av} is a function of them, so that for our assumption of an arbitrary V_{av}, the variations of I_k with A_T, R, Θ, σ_i and $\sqrt{\Delta \varepsilon_{ai}}$ are generally the same as those of P_h.

It should be observed that range has a very significant effect on P_h and I_k.

$$\frac{I_{K_1}}{I_{K_2}} - \left(\frac{R_2}{R_1}\right)^{(2-3)}$$

Therefore, training and equipment plans should consider the very considerable gains to be obtained by firing at short range. (For example, although aircraft armor to protect against small-arms anti-aircraft fire and fragment damage would increase W_o, the reduction in safe firing range resulting from its installation might well increase P_h sufficiently to result in higher I_k's for the armored airplanes.)

For a particular weapon, where V_{av} may not be independent of R, V_a, and Θ, the same general trends follow, modified as indicated in the above expression for σ_g.

The above analysis has been based upon an approximation for the projectile trajectory. The errors introduced by this approximation should be explored. First it is necessary to obtain expressions for the average velocities and their derivatives for the various weapons. These follow:

For bombs:

$$V_{av} \doteq V_a \left(\text{within } 1.10\, V_a > V_{av} > V_a \begin{cases} R \leq 6000 \text{ ft.} \\ V_a \geq 500 \text{ f.p.s.} \end{cases}\right)$$

For rockets:

$$V_{av} \doteq \frac{R\,(V_a + V_b)}{R + V_b t_b/2};\ \left((V_a + V_b/2)\, t_b < R\right)\left(\text{within } \frac{R}{t_F} < V_{av} < 1.10\,\frac{R}{t_F};\ R \leq 600 \text{ ft.}\right)$$

$$\doteq \frac{V_a}{2}\left[1 + \sqrt{1 + \frac{2 V_b R}{V_a^2 t_b}}\right];\ \left((V_a + V_b/2)\, t_b > R\right)$$

For guns:

$$V_{av} \doteq V_a + V_m \left(\text{within } \frac{R}{T_F} < V_{av} < 1.10\,\frac{R}{T_F};\ R \leq 6000 \text{ ft.}\right)$$

Then:

For bombs:

$$\sigma_R \doteq \frac{\Delta R}{R}\,\frac{g}{2}\,\frac{R\cos\theta}{V_a^2}$$

$$\sigma_{V_a} = \frac{-2\Delta V_a}{V_a}\,\frac{g}{2}\,\frac{R\cos\theta}{V_a^2}$$

$$\sigma_\theta = \frac{-\Delta\theta}{\theta}(\theta\tan\theta)\,\frac{g}{2}\,\frac{R\cos\theta}{V_a^2}$$

For rockets:

$$\sigma_R \doteq \frac{\Delta R}{R}\left[1-\left(\frac{V_b t_b}{2R}\right)^2\right]\frac{g}{2}\,\frac{R\cos\theta}{(V_a+V_b)^2}\quad\left(\frac{V_b t_b}{2R} < 1-\frac{V_a}{R}\right)$$

$$\sigma_{V_a} = -2\,\frac{\Delta V_a}{V_a}\,\frac{V_a}{(V_a+V_b)}\left(1+\frac{V_b t_b}{2R}\right)^2\frac{g}{2}\,\frac{R\cos\theta}{(V_a+V_b)^2}\quad\left(\frac{V_b t_b}{2R} < 1-\frac{V_a}{R}\right)$$

$$\sigma_\theta \doteq -\frac{\Delta\theta}{\theta}(\theta\tan\theta)\left(1+\frac{V_b t_b}{2R}\right)^2\frac{g}{2}\,\frac{R\cos\theta}{(V_a+V_b)^2}$$

For guns:

$$\sigma_R = \frac{\Delta R}{R}\,\frac{g}{2}\,\frac{R\cos\theta}{(V_a+V_m)^2}$$

$$\sigma_{V_a} = \frac{2\Delta V_a}{V_a}\,\frac{V_a}{(V_a+V_m)}\,\frac{g}{2}\,\frac{R\cos\theta}{(V_a+V_m)^2}$$

$$\sigma_\theta = -\frac{\Delta\theta}{\theta}(\theta\tan\theta)\,\frac{g}{2}\,\frac{R\cos\theta}{(V_a+V_b)^2}$$

Then, assuming that the errors are independent:

For bombs:

$$\sigma_g = \frac{g}{2}\,\frac{R\cos\theta}{V_a^2}\,\frac{\Delta R}{R}\sqrt{1+4\left(\frac{\Delta V_a}{V_a}\,\frac{R}{\Delta R}\right)^2+\theta^2\tan^2\theta\left(\frac{\Delta\theta}{\theta}\,\frac{R}{\Delta R}\right)^2}$$

For rockets:

$$\sigma_g = \frac{g}{2}\,\frac{R\cos\theta}{(V_a+V_b)^2}\,\frac{\Delta R}{R}\sqrt{\left[1-\left(\frac{V_b t_b}{2R}\right)^2\right]^2+\left(1+\frac{V_b t_b}{2R}\right)^4\left[4\left(\frac{V_a}{V_a+V_b}\right)^2\left(\frac{\Delta V_a}{V_a}\,\frac{R}{\Delta R}\right)^2 + \theta\tan^2\theta^2\left(\frac{\Delta\theta}{\theta}\,\frac{R}{\Delta R}\right)^2\right]}$$

For guns:

$$\sigma_g = \frac{g}{2}\,\frac{R\cos\theta}{(V_a+V_m)^2}\,\frac{\Delta R}{R}\sqrt{1+4\left(\frac{V_a}{V_a+V_m}\right)^2\left(\frac{\Delta V_a}{V_a}\,\frac{R}{\Delta R}\right)^2+\theta^2\tan^2\theta\left(\frac{\Delta\theta}{\theta}\,\frac{R}{\Delta R}\right)^2}$$

If it is assumed that $\dfrac{\Delta R}{R} = \dfrac{\Delta V}{V} = \dfrac{\Delta\theta}{\theta}$

For bombs:

$$\sigma_g = \frac{g}{2}\,\frac{R\cos\theta}{V_a^{\,2}}\,\frac{\Delta R}{R}\sqrt{5 + \theta^2\tan^2\theta}$$

For rockets:

$$\sigma_g = \frac{g}{2}\,\frac{R\cos\theta}{(V_a+V_b)^2}\,\frac{\Delta R}{K}\sqrt{\left[1-\left(\frac{V_b t_b}{2R}\right)^2\right]^2+\left(\frac{1+V_b t_b}{2R}\right)^4\left[4\left(\frac{V_a}{V_a+V_b}\right)^2+\theta^2\tan^2\theta\right]}$$

For guns:

$$\sigma_g = \frac{g}{2}\,\frac{R\cos\theta}{(V_a+V_m)^2}\,\frac{\Delta R}{R}\sqrt{1+4\left(\frac{V_a}{V_a+V_m}\right)^2+\theta^2\tan^2\theta}$$

Figure 4 plots σ_g versus R_1 ($\frac{\Delta R}{R} = \frac{\Delta V_a}{V_a} = \frac{\Delta\theta}{\theta} = 0.005$) for various values of θ, V_b, and V_m corresponding to existing bombs, rockets, and guns. For rockets and bombs, V_a was set equal 500 ft./sec.; for guns, V_a was set equal to zero. Actual values of σ_g as taken from trajectory Table I are shown for comparison. It will be observed that the calculated rocket deviations correspond quite well to the actual, the errors ranging from 2 to 7 percent high for the bomb, 2 to 12 percent low for the 5" AR, 14 to 26 percent low for the 5" HVAR, and 8 to 13 percent low for the gun. These errors are, in most cases, no greater than those which must result from assumptions for σ_i. It appears then that trends shown by the approximate analytic expressions derived above will be generally correct and that the absolute values will not be greatly in error.

σ_i

σ_i has been defined as dispersions due to all causes other than release error along the sight line. These would include free flight ballistic dispersions, dispersions caused by mechanical and aerodynamic disturbances, sighting errors, alignment errors and azimuth errors.

It will be assumed that these errors are circular. The values given below are for the linear components ($\sigma_{i_x} = \sigma_{i_y} = 0.707\sigma_i$ (circ)). The free flight ballistic dispersions have been given in other EngOrd reports (2,3,4,5) and are approximately:

3-4 mils - Bombs (Existing bombs with modified fin assemblies and proposed new family of bombs)

3-7 mils - Rockets (Air-fired, fin-stabilized)

1-3 mils - Guns (Air-fired)

There have been no satisfactory isolations of the other dispersions contained in σ_i. Therefore, for the remainder of this analysis two assumptions as to its value will be made. The first (lower limit) will be that σ_i is equal to the ballistic dispersion alone, value to be:

4 mils - Bombs

4 mils - Rockets

2 mils - Guns

The second (upper limit) will be that σ_i is equal to dispersions generally found in firing tests corrected for σ_y. These values are approximately (6).

9 mils - Bombs

9 mils - Rockets

5 mils - Guns

<u>Release Error Control Requirements</u>

With the aid of the above equations and numbers it is possible to approximate the values within which Δ R, Δ V, and $\Delta\theta$ must be maintained in order that dispersion along the sight line, σ_x, will approach its minimum practical limit. Since

$$\sigma_x = \sqrt{\sigma_i^2 + \sigma_g^2}$$

the minimum limit will be σ_i, with Δ R, Δ V, $\Delta\theta$ all equal to zero.

However, it will be assumed that a minimum limit below which further expenditure of effort to reduce release errors would be impractical will be:

$$\sigma_x \leq 1.2\sigma_i, \text{ or}$$

$$\sqrt{\sigma_i^2 + \sigma_g^2} \leq 1.2\sigma_i$$

$$\sigma_i^2 + \sigma_g^2 \leq 1.44\sigma_i^2$$

$$\sigma_g \leq 0.663\sigma_i$$

Then for bombs and rockets:

$$\sigma_g \leq \begin{matrix} 6 \text{ mils } (\sigma_i = 9 \text{ mils}) \\ 2.7 \text{ mils } (\sigma_i = 4 \text{ mils}) \end{matrix} \quad \sigma_x \begin{matrix} = 11 \text{ mils} \\ = 5 \text{ mils} \end{matrix}$$

For guns:

$$\sigma_g \leq \begin{matrix} 3.3 \text{ mils } (\sigma_i = 5 \text{ mils}) \\ 1.3 \text{ mils } (\sigma_i = 4 \text{ mils}) \end{matrix} \quad \sigma_x \begin{matrix} = 6 \text{ mils} \\ = 2.4 \text{ mils} \end{matrix}$$

Then using the approximate equation for σ_g:

$$.663 \geq \sigma_g \doteq \frac{g}{2}\frac{R\cos\theta}{V_{av}^2}\sqrt{\left(\frac{\Delta R}{R}\right)^2 + \left(2\frac{\Delta V_a}{V_{av}}\right)^2 + (\Delta\theta\tan\theta)^2}$$

$$1.326\frac{\sigma_i V_{av}^2}{g R\cos\theta} \doteq \geq \sqrt{\left(\frac{\Delta R}{R}\right)^2 + \left(2\frac{\Delta V_a}{V_{av}}\right)^2 + (\Delta\theta\tan\theta)^2} = \Delta^2$$

Using the above equation, the maximum allowable value of Δ^2 may be approximated. Then it is necessary to assign maximum allowable value to $\frac{\Delta R}{R}$, $\frac{\Delta V_a}{V_a}$, and $\Delta\theta$.

First, it will be assumed that there are maximum limits within which these errors can be controlled by even a very simple fire control system (fixed sight, standard release range, trained pilot). Δ R is probably the most difficult error to estimate or control; ΔV_a somewhat less difficult; and $\Delta\theta$ is the least difficult. Experiences of trained gunnery pilots, firing with fixed sights indicate that these errors can be held within the following maximum limits:

$\Delta R = 0.5\, R$ $\qquad \Delta V_a = 50$ ft./sec. $\qquad \Delta\theta = 5^{\circ}$

It will be assumed that no errors greater than these will be permitted. Then the necessary reductions below these limits to satisfy the above equation will be determined. The distribution of errors will be such that when all the allowable maximum errors are below the simply controllable maximum limits,

$$\frac{\partial \sigma_i}{\partial\left(\frac{\Delta R}{R}\right)} = \frac{\partial \sigma_i}{\partial\left(2\frac{\Delta V_a}{V_{av}}\right)} = \frac{\partial \sigma_i}{\partial(\Delta\theta \tan\theta)}$$

With these assumptions, the maximum allowable values have been computed for a range of V_{av}'s from 500 to 3000 ft./sec., $R = 3000$ and 6000 feet, $\theta = 20^{\circ}$ and 60°, $\sigma_0 = 2$, 5, and 9 mils and $V_a = 500$ ft./sec. The results are presented in Table I. A generalized summary of the results is given below, for $\sigma_1 = 5$ mils.

Weapon	Range (ft.)	Allowable Errors		
		$\Delta R/R$	$\Delta V_a/V_a$	θ
Guns (V_m 1500 ft./sec.)	6000	0.15	0.1	5°
	3000	0.3	0.1	5°
Rockets				
Rockets (V_m 1000 ft./sec.)	6000	0.05	0.1	5°
	3000	0.15	0.1	5°
Rockets (500 V_m 1000 ft./sec.)	6000	0.02	0.02	2°
	3000	0.05	0.05	5°
Bombs	6000	0.005	0.003	1°
	3000	0.01	0.006	1°

Therefore, if the effect of gravity drop is to be reduced to a minimum practical limit, range error must be controlled within limits from $\frac{\Delta R}{R} <$ 0.005 for bombs to $\frac{\Delta R}{R} < 0.15$ for guns. Velocity and range error effects, for errors below those controllable by fairly simple systems, are negligible, except in the case of bombing.

Figures 4a through 4e show the effects of release error on σ_1 for various weapons at various ranges and release angles.

$\underline{P_h}$

Finally then, the following expressions are derived for P_h.

For bombs:

$$P_h \doteq \frac{A_T V_a^2}{\pi g R^3 \cos\theta} \cdot \frac{1}{\sigma_{i_B}\frac{\Delta R}{R}\sqrt{1+\left(\frac{2V_a^2\sigma_{i_B}}{gR\cos\theta}\right)^2\left(\frac{R}{\Delta R}\right)^2+4\left(\frac{\Delta V_a}{V_a}\frac{R}{\Delta R}\right)^2+\theta^2\tan^2\theta\left(\frac{\Delta\theta}{\theta}\frac{R}{\Delta R}\right)^2}}$$

$$(4<\sigma_{i_B}<9\text{ mils})(V_a \geq 500\text{ fps})(R \leq 6000\text{ ft.})(P_h \leq .5)$$

For rockets:

$$P_h \doteq \frac{A_T(V_a+V_b)^2}{\pi g R^3\cos\theta}\cdot\frac{1}{\sigma_{i_R}\left[1-\left(\frac{V_b t_b}{2R}\right)^2\right]\frac{\Delta R}{R}\sqrt{1+\frac{\left[\frac{2(V_a+V_b)^2\sigma_{i_R}}{gR\cos\theta}\right]^2}{\left[1-\left(\frac{V_b t_b}{2R}\right)^2\right]^2}\left(\frac{R_t}{\Delta R}\right)^2+\left[\frac{1+\frac{V_b t_b}{2R}}{\frac{V_b t_b}{2R}}\right]^2\left[4\left(\frac{V_a}{V_a+V}\right)^2\left(\frac{\Delta V_a}{V_a}\frac{R}{\Delta R}\right)^2+\theta^2\tan^2\theta\left(\frac{\Delta\theta}{\theta}\frac{R}{\Delta R}\right)^2\right]}}$$

$$(4<\sigma_{i_a}<9\text{ mils})$$
$$((V_a+\sigma_b/2)t_b<R)$$
$$(R\leq 6000\text{ ft.})$$

For guns: $(P_h \leq .5)$

$$P_h \doteq \frac{A_T(V_a+V_m)^2}{\pi g R^3\cos\theta}\cdot\frac{1}{\sigma_{i_g}\frac{\Delta R}{R}\sqrt{1+\left[\frac{2(V_a+V_m)^2}{gR\cos\theta}\sigma_{i_g}\right]^2\left(\frac{R_t}{\Delta R}\right)^2+4\left(\frac{V_a}{V_a+V_m}\right)^2\left(\frac{\Delta V_a}{V_a}\frac{R}{\Delta R}\right)^2+\theta^2\tan\theta\left(\frac{\Delta\theta}{\theta}\frac{R}{\Delta R}\right)^2}}$$

$$(2<\sigma_{i_g}<5\text{ mils})$$
$$(R\leq 6800\text{ ft})$$
$$(P_h \leq .5)$$

Figures 5a through 5d show the values of $\frac{P_h}{A_T}$ for various weapons, dive angles, and the limiting values of σ_i, plotted as a function of range. Figures 6a and 6b show P_h/A_T plotted as a function of V_b for rockets, and V_m for guns. The marked increase of hit probability with decreasing range will be noted in Figure 5. It will be noted that no significant improvements in rocket hit probabilities will be made until reductions in inherent dispersions are accomplished. However, improvements in bombing fire control above those assumed will result in significant improvements in bombing hit probabilities. Figure 6 indicates that as long as fire control errors remain large, burnt or muzzle velocities should be kept high to improve hit probabilities. As fire control errors are reduced, velocities may be correspondingly reduced. It is obvious, that with perfect fire control, velocity will have no effect on hit probability. Similarly as long as inherent dispersions are high, burnt or muzzle velocities may remain relatively low. As inherent dispersions are reduced, velocities should be increased to improve hit probabilities.

In Figure 5, the hit probabilities obtained in Air Proving Ground tests with 5" HVAR rockets, using the A-1CM sight with varying degrees of sensitivity has been shown (7). Ranges were uncertain, between 2500 and 3500 feet. Results were in remarkable agreement with those hypothesized with the use of the foregoing approximations.

Based on the data of Figure 5, Figure 7 shows the number of rounds required per hit (subject to the errors inherent in use of the approximate formula) as a function of range and target area for guns, rockets, and bombs and for the combinations of maximum range errors and inherent dispersion and minimum range errors and inherent dispersions. The maximums may be considered as approximating present systems, the minimums as the limit of inherent improvement. Three target areas have been chosen, 200 sq. ft. (approximately that of a tank, side on), 2000 sq. ft. (a pill-box or artillery emplacement) and 20,000 sq. ft. (troop vehicles or supply concentration).

In analyzing these data, let us consider that a maximum of 8 rounds per hit are desired. Then the following table indicates the maximum ranges in feet at which the various weapons may be used.

Target Area	Guns		Rockets		Bombs	
	Present Errors	Improved	P.E.	Imp.	P.E.	Imp.
200 sq. ft.	$R < 3000$ ft.	$R < 6000$	$R < 1500$	$R < 3000$	$R < 1000$	$R < 1500$
2000 sq. ft.	$R < 6000$	$R < 6000$	$R < 4000$	$R < 6000$	$R < 2000$	$R < 3500$
20,000 sq. ft.	$R < 6000$	$R < 6000$	$R < 6000$	$R < 6000$	$R < 4500$	$R < 6000$

On the basis of hit probabilities above, with specifications as established, guns should be used for the 200 sq. ft. target, bombs should not be used, and extremely close ranges are necessary with rockets. Either guns or rockets might be used against the intermediate area target, but close ranges are necessary with bombs. Guns, rockets, or bombs might be used against the 20,000 sq. ft. target.

$\frac{W_h}{W_o}$ - Ratio of Total Warhead Weight to Armament System Weight

The total warhead weight carried by the airplane:

$$W_h = N_R w_h$$

The total armament system weight:

$$W_o = W_R + W_g + W_i + W_e$$

$$W_R = N_R W_R$$

$$W_g = N_g W_g$$

where:

W_o = Armament system weight

W_h = Total warhead weight

W_R = Total round weight

W_i = Total installation weight

W_e = Total fire control weight

W_g = Total gun weight

N_R = Number of rounds

N_g = Number of guns

w_h = Warhead weight per round

w_R = Round weight

w_g = Gun weight

So:
$$\frac{W_h}{W_o} = \frac{1}{\dfrac{N_R W_R + N_g w_g + W_i + W_e}{N_R w_h}}$$

$$= \frac{1}{\dfrac{W_R}{w_h} + \dfrac{N_g}{N_R}\dfrac{w_g}{w_h} + \dfrac{W_i}{N_R w_h} + \dfrac{W_e}{N_R w_h}}$$

In other EngOrd reports (2, 3, 4, 5, 9) expressions have been derived for $\frac{W_R}{w_h}$ $\frac{W_g}{w_h}$ and W_i as functions of round diameter, rate of fire, velocity of round, and subsidiary parameters characteristic of the type of weapon. However, there are no such expressions derived for $\frac{W_e}{N_R w_h}$. Therefore, in this report, it will be considered that comparable fire control systems are provided for all weapons (weapons will be compared for the same release errors), W_e will be included in the basic airplane weight, and the term eliminated from the above expression.

Bombs

For bombs, gun weight is zero, and the installation weight per round varies roughly linearly with the weight of the round. Thus

$$\frac{w_g}{w_h} = 0$$

and from (2),
$$\frac{W_i}{N_R w_h} \doteq 0.04$$
$$\frac{W_R}{w_h} \doteq 1.05$$

(considering warhead weight to be the total bomb weight minus the stabilizing system weight)

So that:

$$\frac{w_h}{W_o} \doteq \frac{1}{1.05 + 0.04} = 0.92$$

Rockets

As for bombs, gun weight is zero, and the installation weight per round varies roughly linearly with the weight of the round. Thus

$$\frac{w_g}{w_h} = 0$$

and from (3),

$$\frac{W_i}{N_R w_h} = \begin{cases} .10 \text{ (pylon launcher)} \\ .50 \text{ (packaged launcher)} \end{cases}$$

$$\frac{W_R}{w_h} \doteq \frac{e^{V_b/6800}}{1 + w_h/w_p\left(1 - e^{V_b/6800}\right)}$$

$$\frac{W_m}{w_p} = \begin{cases} .15 + 2.7/D_m \text{ (new rockets (5" HPAG, 2".75 AAFFR, 8 cm OER))} \\ 3.0 + 2.7/D_m \text{ (std. W.W. II rockets (5" HVAR, 3".5 AR, 11".75 AR))} \end{cases}$$

D_m = Motor diameter, inches

V_b = Burnt velocity, relative to launcher, ft./sec.

$\frac{W_m}{W_p}$ = Ratio of rocket motor weight (burnt) to propellant weight.

Then:

$$\frac{W_h}{W_o} \doteq \frac{1}{\dfrac{e^{V_b/6800}}{1 + \frac{W_m}{W_p}\left(1 - e^{V_b/6800}\right)} + K}$$

$$\doteq \frac{1}{\dfrac{1 + V_b/6800}{1 - \frac{W_m}{W_p}\frac{V_b}{6800}}} + K \quad \left(500 < V_b < 2500\right)$$

$$\doteq \frac{6800 - W_m/W_p\, V_b}{K\left(6800 - \frac{W_m}{W_p} V_b\right) + \left(6800 + V_b\right)}$$

For simplicity, say

$$\frac{W_i}{N_R W_h} = K \doteq 0.3$$

And that

$$\frac{W_m}{W_p} \doteq \begin{cases} 1.0 \text{ (new rocket)} \\ 3.0 \text{ (older std. rocket)} \end{cases}$$

Then: $$\frac{w_h}{W_o} = \frac{6800 - V_b}{8840 + 0.7 V_b} \text{ (new rocket)}$$
$$= \frac{6800 - 3V_b}{8840 + 0.1 V_b} \text{ (older std rocket)}$$

These values are plotted in Figure 8. Actual values are shown for comparison. Good agreement is shown between the estimated and actual values for the older rockets but probably will be approached as development continues. It will be noted that $\frac{w_h}{W_o}$ decreases markedly with increasing V_b.

Standard Guns

From (4,5):

$$\frac{W_g}{w_h} \doteq \frac{V_m^2}{D_h^{.41}}\left[19.1 + .0455 r\right] \times 10^{-6}$$

where

V_m = Muzzle velocity, ft./sec.

D_h = Projectile diameter, inches

r = Rate of fire, rounds per minute

$$\frac{W_R}{w_h} \doteq \frac{.136 V_m^2 + 10^{-6}}{D_h^{.41}} + 1.2$$

$$\frac{W_i}{N_R w_h} \doteq \frac{.9 w_g}{N_R w_h}$$

so that:

$$\frac{w_h}{W_o} = \frac{1}{\left[\frac{.136 V_m^2 \times 10^{-6}}{D_h^{.41}} \times 1.2\right] + \frac{N_g}{N_R}\frac{V_m^2}{D_h^{.41}}\left[19.1 + .0455 r\right] \times 10^{-6}\left[1.9\right]}$$

$$= \frac{D_h^{.41} \times 10^{-6}}{V_m^2\left[.136 + 1.9\frac{N_g}{N_R}\left[19.1 + .0455 r\right]\right] + 1.2\, D_h^{.41} \times 10^{-6}}$$

For example, let us consider two classes of guns, a small caliber, high cyclic rate gun (say 20 mm., 650 rpm) and a large caliber, low cyclic rate gun (say 75 mm., 10 rpm). Then

$$\frac{w_h}{W_o}(20\,mm) = \frac{.906 \times 10^6}{V_m^2\left[.136 + 92.5\frac{N_g}{N_R}\right] + 1.09 \times 10^6}$$

$$\frac{w_h}{W_o}(75\,mm) = \frac{1.553 \times 10^6}{V_m^2\left[.136 + 37.2\frac{N_g}{N_R}\right] + 1.863 \times 10^6}$$

Values are plotted in Figure 9, and compared against actual values. Good agreement is found. Again, it is found that $\frac{w_h}{W_o}$ decreases markedly with increasing muzzle velocity. It will be noted that a number of rounds per gun has a significant effect on $\frac{w_h}{W_o}$, the larger number of rounds per gun giving higher $\frac{w_h}{W_o}$ ratios. This would be expected until the total weight of the rounds are equal to or greater than the total weight of the gun. Also, inspection of the above equations reveals that for the same muzzle velocities and rounds per gun, increases in rates of fire decreases $\frac{w_h}{W_o}$.

<u>Recoilless Guns</u>

From (5 and 8):

$$\frac{w_g}{w_h} = 9.5 \times 10^{-6} V_m^2 \quad \text{(existing designs)}$$
$$\phantom{\frac{w_g}{w_h}} = 4.8 \times 10^{-6} V_m^2 \quad \text{(proposed future designs)}$$

(not including automatic feed)

$$\frac{w_R}{w_h} = \frac{w_h + W_c + w_p}{w_h} = 1 + \frac{W_c + w_p}{w_h}$$

$$w_p = 1.46 \times 10^{-4}\left(\frac{w_h}{2g} V_m^2\right)^{.795} \quad \text{(57-105 mm. guns)}$$

$$\frac{w_p}{w_h} = 6.78 \times 10^{-6} \frac{V_m^{1.59}}{D_h^{.615}}$$

$$W_c = 0.7\, w_p$$

For an automatic feed mechanism, we shall assume (based on the standard gun) $\frac{w_f}{w_h} \doteq .0455r\frac{V_m^2}{D_h^{.41}} \times 10^{-6} + .1\frac{w_g}{w_h}$

$$= V_m^2 \times 10^{-6}\left[.95 + \frac{.0455r}{D_h^{.6}}\right]$$

(existing design)

$$= V_m^2 \times 10^{-6}\left[.48 + \frac{.0455r}{D_h^{.6}}\right]$$ (proposed new design)

Then:

$$\frac{W_g}{w_h} \doteq V_m^2 \times 10^{-6}\left[10.45 + \frac{.0455r}{D_h^{.6}}\right]$$ (existing design)

$$\doteq V_m^2 \times 10^{-6}\left[5.28 + \frac{.0455r}{D_h^{.6}}\right]$$ (proposed new design)

$$\frac{W_R}{w_h} \doteq 1 + 11.5 \times 10^{-6}\frac{V_m^{1.59}}{D_h^{.6}}$$

where:

w_f = Weight of automatic feed mechanism

w_c = Weight of care of round

The installation weight should be between those for rockets and standard guns, say

$$\frac{W_i}{N_R w_h} \doteq .5\frac{W_g}{N_R w_h}$$

Then:

$$\frac{w_h}{W_o} \doteq \frac{1}{1 + V_m^2 \times 10^{-6}\left[\frac{11.5}{D_h^{.6} V_m^{.41}} + 1.5\frac{N_g}{N_R}\left(10.45 + \frac{.0455r}{D_h^{.6}}\right)\right]}$$ (existing guns)

$$\doteq \frac{1}{1 + V_m^2 \times 10^{-6}\left[\frac{11.5}{D_h^{.6} V_m^{.41}} + 1.5\frac{N_g}{N_R}\left(5.28 + \frac{.0455r}{D_h^{.6}}\right)\right]}$$ (proposed guns)

Let us now consider a large caliber, low cyclic rate gun (say 75 mm., 10 rpm). Then

$$\frac{w_h}{W_o} \doteq \frac{1}{1 + V_m^2 \times 10^{-6}\left[\frac{6.25}{V_m^{.41}} + 16\frac{N_g}{N_R}\right]}$$ (existing guns)

$$\doteq \frac{1}{1 + V_m^2 \times 10^{-6}\left[\frac{6.25}{V_m^{.41}} + 8.28\frac{N_g}{N_R}\right]}$$ (proposed guns)

Values are plotted in Figure 10. The same trend of marked decrease of $\frac{w_h}{W_o}$ with increase in V_m is found as with guns. The significance of the number of rounds per gun will again be noted.

Gun-Launched Rockets

Inasmuch as gun-launched rockets (or closed-breech rocket launchers) are still in early development stages, there are little statistical data against which empirical relationships describing the family can be checked. In general, it would appear that the weight of the launcher must be increased over that for the pure rocket as a function of the rate of fire and velocity upon leaving the launcher, as in the case of guns. The ratio of rocket warhead weight to round weight would be expected to be less than for the pure rocket because of the necessity for strengthening the case to withstand the higher initial accelerations. Therefore, the over-all $\frac{W_h}{W_o}$ for the gun-launched rocket would be less than for the pure rocket. Counterbalancing this, however, is the increase in accuracy, the dispersions approaching those of guns.

For the T131 rocket, used with the T110E2 launcher

w_h = 5.2 lb.

w_g = 300 lb. (launcher, magazine, and feed weight)

w_R = 10.7 lb.

V_b = 2500 ft./sec.

N_R = 25

N_{gr} = 650 rpm

Assuming an installation weight equal to 0.5 w_g

$$\frac{W_h}{W_o} = 0.182 \text{ @ } V_b = 2500 \text{ ft./sec.}$$

This compares with the values of 0.144 found for the pure rockets, (light case), 0.038 for a standard gun with the same performance, or 0.095 for a recoilless gun with the same performance.

Comparison of $\frac{W_h}{W_o}$ for Various Weapons

Of the weapons described, the bomb has the highest value of $\frac{W_h}{W_o}$

because no propellant, or structure to resist the supporting forces is necessary. The rocket (except for the closed breech launched rocket) requires a comparatively large amount of propellant, but relatively little structure. The recoilless gun requires less propellant than the rocket, but more structure. The gun requires even less propellant, but even more structure. This will be noted in the comparison of warhead weight to round weights where the rocket has the lowest ratio of the three, the recoilless gun an intermediate ratio, and the gun the lowest ratio. Therefore, at some number of rounds per gun, the $\frac{W_h}{W_o}$ ratios of the three weapons should be the same, that weapon requiring the greatest structural weight requiring the most rounds per gun. These numbers are tabulated below, the numbers corresponding to the number of rounds per gun which must be carried to give an $\frac{W_h}{W_o}$ equal to that of a rocket, the gun muzzle velocities being equal to the rocket's burnt velocity. An intermediate velocity of 1500 feet per second was chosen for this comparison.

Projectile Diameter	Number of Rounds per Gun Required at		
	10 rpm	600 rpm	
1 inch	235	550	
2 inch	145	345	Standard guns
5 inch	90	215	
1 inch	---	---	
2 inch	2095	875	Recoilless Guns
5 inch	50	145	

A similar table based on the number of rounds per gun required to equal the $\frac{W_h}{W_o}$ of the T131 rocket and launcher, 2.75 projectile, 650 rpm, 25 rounds per launcher and 2500 feet per second burnt velocity is given below.

Projectile Diameter	Number of rounds per gun required at 650 rpm	
2.75	105	Standard Gun
2.75	70	Recoilless Gun

Values of L, The Aircraft Armament Logistic Factor for Various Weapons.

As previously defined, the aircraft armament logistic factor is the product of the reciprocal of the probability of hit and the ratio of total weight of armament system carried to total weight of warhead carried. Thus:

$$L_w = \frac{1}{P_h}\frac{W_o}{w_h}$$

or

$$\frac{1}{L_w} = P_h \frac{w_h}{W_o}$$

P_h and $\frac{w_h}{W_o}$ are mutually related by their dependence on muzzle (gun) or burnt (rocket) velocities. Their variations as functions of velocity have been individually discussed in previous sections. The over-all variation will be discussed below.

Bombs

For bombs $\frac{w_h}{W_o}$ is essentially fixed. Therefore $1/L_w$ varies linearly with the probability of hit. Relative values of the product of L_w and the target area are given in the following table:

$L_w A_T$

R →	3000		6000	feet
σ_L →	4	9	4	9 mils
$\Delta R/R$				
0.1	9500	21,500	74,000	180,000
0.5	22,000	49,000	175,000	390,000

Rockets

The variation of $1/L_W A_T$ for rockets as a function of range, σ_1, burnt velocity and range release error is plotted in Figure 11. It will be noted that $1/L_W$ reaches a maximum (L_W reaches a minimum) within the velocity range of the rocket. As might be expected, the optimum velocity is somewhat lower for small release errors than for high; somewhat lower for large inherent dispersions than for small; and somewhat lower for short ranges than for long. The best compromise velocities (weighted toward firing at long ranges), appear to be approximately 1000 feet per second for the older (higher case weight) rockets, and 1600 feet per second for the newer (low case weight) rockets. Both values are somewhat lower than those found in existing designs. Relative values of the product of $L_W A_T$ are shown below:

$R \rightarrow$		$L_W A_T$ 3000		6000		feet
$\frac{\Delta R}{R}$	$\sigma_1 \rightarrow$	4	9	4	9	mils
0.1		2600	11,000	13,500	52,500	Older rockets
0.5		5350	15,500	43,500	95,000	
0.1		1650	9100	8150	35,500	Newer rockets
0.5		2650	9800	18,500	50,000	

Standard Guns

The variation of $1/L_W A_T$ for standard guns, as a function of range, σ_1, muzzle velocity, and range release error are plotted for two guns (a small caliber high cyclic rate, and a large caliber, low cyclic rate gun, for various numbers of rounds per gun) in Figure 12. There, optimum velocities are also shown to be lower than standard design, approximately

1700 feet per second for both types. The trends of variation of optimum velocity with release error, inherent dispersions, and range are the same as with rockets. Relative values of the product of $L_W A_T$ are shown below.

$L_W A_T$

$R \rightarrow$	3000		6000	feet		
$\sigma_i \rightarrow$	2	5	2	5	mils	
$\frac{\Delta R}{R}$						
0.1	785	4550	4000	20,000	200 rounds per gun	Small caliber high cyclic rate
0.5	1700	6650	12,000	33,500		
0.1	600	3500	3250	15,000	400 rounds per gun	
0.5	1200	4750	9500	23,500		
0.1	1550	14,000	11,500	30,000	10 rounds per gun	Large caliber low cyclic rate
0.5	4650	15,500	35,500	85,000		
0.1	1250	7050	6150	29,500	20 rounds per gun	
0.5	2550	9250	20,000	51,500		

It will be noted that for corresponding inherent dispersions, in the ranges of rounds per gun shown, the logistic factor for guns is higher than for the newer rockets (effectiveness per pound of installation weight is lower). This was indicated in the comparison given in the section discussing $\frac{W_h}{W_o}$. The lower limiting values for guns than for rockets are due to the lower range of dispersions.

Recoilless Guns

The variation of $1/L_W A_T$ for recoilless guns, as a function of range, σ_i, muzzle velocity and range release error are plotted in Figure 13 for two gun designs, one corresponding to existing practice, and a lighter one corresponding to proposed new designs, both taken as a large caliber, low cyclic rate weapon. The optimum velocity is shown to be approximately 1200 feet per second (weighted toward the longer range firing), for present design and approximately 1400 feet per second for the new design. The trends of variation of optimum velocity with release error, inherent dispersions, and range are the same as with rockets and standard guns. Relative values of the product of L_w and the target area, for the above velocities are shown below:

$L_w A_T$

R →	3000		6000		feet	
σ_i →	2	5	2	5	mils	
$\Delta R/R$ ↓						
0.1	1150	3330	7350	28,000	10 rounds per gun	Present Designs
0.5	3300	9100	41,500	111,000		
0.1	800	2300	5100	18,000	20 rounds per gun	
0.5	2300	6600	18,000	49,000		
0.1	900	4750	5250	22,000	10 rounds per gun	New Designs
0.5	2300	6650	18,000	45,500		
0.1	685	3600	3900	16,000	20 rounds per gun	
0.5	1750	5400	13,500	26,500		

It will be noted that for recoilless guns, as with standard guns, in the ranges of rounds per gun shown, for corresponding inherent dispersions, the logistic factor is higher than for the newer rockets, as was indicated in the $\frac{W_H}{W_o}$ comparison. The lower limiting values for recoilless guns than for rockets are due to the lower range of dispersions. However, the logistic factors for large caliber low cyclic rate recoilless guns are smaller than those for corresponding standard guns under comparable conditions.

Gun-Launched Rockets

The logistic factor for gun-launched rockets can be confidently determined only at the design velocity of the existing weapon, the T131 at 2500 feet per second. Assuming its dispersions to be in the same range as guns, the relative values of the product of $L_W A_T$ are given below.

$L_W A_T$

$R \rightarrow$	3000		6000	
$\sigma_l \rightarrow$	2	5	2	5
$\Delta R/R$				
0.1	1300	7800	5650	32,000
0.5	1950	8550	12,500	42,500

These values are higher than for the recoilless gun (20 rounds per gun) at its best velocity. However, a reduction in the velocity of the T131 would reduce the logistic factors, until, at the same velocity the comparison would be approximately the same as given in the section on $\frac{W_h}{W_o}$, since the same inherent dispersions were assumed both for the recoilless gun and the gun-launched rocket.

Comparison of Weapons on the Basis of L_W.

Figure 14 shows the values of L_W versus range for the following weapons, against three target areas, 200, 2000, and 20,000 square feet.

1. Bomb
2. Rocket - Light case, 1600 ft./sec.
3. Recoilless Guns, Large caliber, 1400 ft./sec.
   a. 10 rpm, 10 rounds per gun
   b. 650 rpm, 20 rounds per gun

4. Standard Guns, 1700 ft./sec.

    a. Large caliber, 10 rpm, 10 rounds per gun

    b. Small caliber, 650 rpm, 600 rounds per gun

5. Gun-Launched Rocket T131, 2500 ft./sec., 650 rpm, 25 rounds per gun

No small caliber recoilless guns are included, since previous examination of logistic factors has indicated their relative inferiority to other weapons. No high cyclic rate large caliber standard guns are included because of the practical difficulties associated with their design and installation, and their relative inferiority in L_w. The projectile velocities are the optimum indicated in the previous sections, except for the T131, whose design velocity was taken.

The target areas, as stated previously, were selected to roughly correspond to the following tactical targets.

200 sq. ft. - Tank, or transport on rail or road

2000 sq. ft. - Pill box, artillery emplacement, or bridge abutment

20,000 sq. ft. - Troop vehicles or supply concentration

In general, the trends follow those shown in the hit probability section, emphasizing the greater weight of the hit probability term in the logistic factor. For the 200 sq. ft. targets at the longer ranges, the better guns have lower logistic factors than rockets, and rockets have lower logistic factors than bombs. For the 2000 sq. ft. targets, the rockets are generally comparable with guns, but better than bombs. For the 20,000 sq. ft. targets, rockets are better than most guns, but at short ranges or low errors and dispersions, bombs are better than all. Above 20,000 feet bombs will be best.

The effect of range is again emphasized, particularly against small targets. Against tanks, effectiveness is gained only by firing at short ranges, no matter what the weapon.

There are some rather important supplements to the conclusions reached in the section on hit probabilities when the weight characteristics of the weapon are considered. The gain in effectiveness from the use of guns over rockets against small targets is not nearly as marked because of the greater weight ratios of the guns. However, the effectiveness of all weapons is so low against small targets that even the small gains in effectiveness made possible by the use of guns should be utilized, since it may make the difference between failure and success of a sortie.

Among the various guns or gun-launched rockets, the small caliber, high cyclic rate, large number of rounds per gun standard gun is superior against all areas. It also has a good logistic factor compared with rockets or bombs. The large caliber standard gun, however, is the least effective of the guns. Between these two lie the gun-launched rocket and the recoilless guns. The gun-launched rocket, for comparable rates of fire and rounds per gun, appears somewhat superior to the recoilless rifle at the longer ranges, higher dispersions, and smaller areas. The two are nearly equal at the shorter ranges, lower dispersions and larger targets.

Examination of Effectiveness Index of Various Weapons

The previous section compared weapons on the basis of logistic factor. However, the results must be modified by the influence of $\left(\frac{S/h}{W_h}\right)T$, the remaining term in the effectiveness index. There are not sufficient effectiveness data to quantitize the effectiveness index in detail. However, the relative index of the different weapons may be examined qualitatively.

Against small targets, the small caliber high cyclic rate gun has the most generally favorable logistic factor. However, $\left(\frac{P(K/h)}{w_h} \right)_r$ for this weapon is essentially zero, when used against armored targets. The low cyclic rate recoilless rifle, or gun-launched rocket, which can deliver larger caliber and weight projectiles would have the best effectiveness index against tanks. The small caliber, high cyclic rate guns would have the best effectiveness index against convoys or trains of vehicles and troops.

Against the intermediate sized targets, unarmored or of light structure, the small caliber gun still shows the best effectiveness index. Against heavy structures, guns or rockets would show approximately equal effectiveness indices, but the lower accelerations of the rocket would enable the more efficient use of a greater number of warheads.

Against targets of 20,000 square foot area and greater, bombs have generally the highest effectiveness index, except where penetration must be accomplished by the kinetic energy of the round rather than by explosive effects.

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1. "Trajectories of Aircraft Rockets", OSRD Report 2540, CIT-UBC 35, January, 1946. (Restricted)

2. "Physical Characteristics of Aircraft Bomb", Vista EngOrd Report 119, California Institute of Technology, December, 1951. (Secret)

3. "Physical Characteristics of Aircraft Rocket", Vista EngOrd Report 120, California Institute of Technology, December, 1951. (Secret)

4. "Weapons Summary of Aircraft Guns", Vista EngOrd Report 115, California Institute of Technology, December, 1951. (Confidential)

5. "Future Development of Aircraft Guns", Vista EngOrd Report 118, California Institute of Technology, December, 1951. (Secret)

6. "Statistical Study of Air-to-Ground Rocketry at the U. S. A. F. Fighter Gunnery Meet", Rand Corporation Report RM-588, April, 1951. (Secret)

7. "Report of the TAC Air Ground Tests with the A-1 Rocket Sight", Air Weapons Research Center, University of Chicago, July, 1951.

8. "Characteristics of Standard Recoilless Guns", Vista EngOrd Report 108, California Institute of Technology, December, 1951. (Confidential)

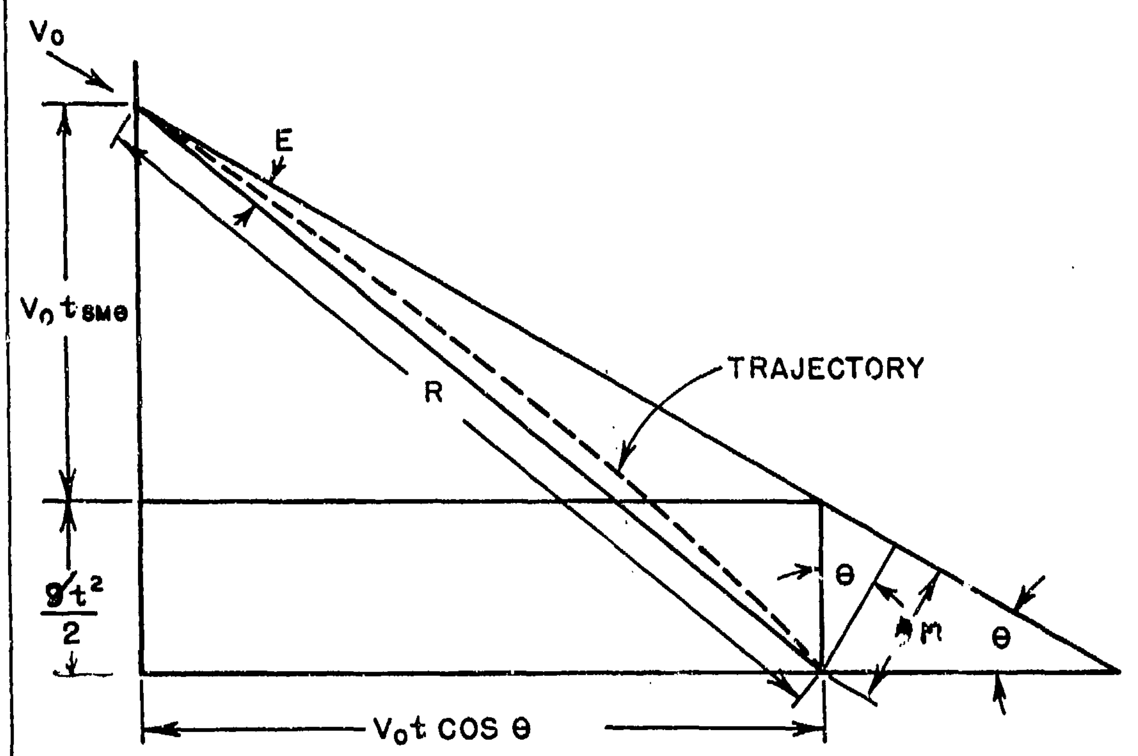
9. "The Ordnance Logistic Factor of the Airplane as a Fire Power Delivery System", Vista EngOrd Report 122, California Institute of Technology, December, 1951. (Secret)

TABLE I

RELEASE ERROR CONTROL REQUIREMENTS*

	σ_i = 2 mils			σ_i = 5 mils			σ_i = 9 mils		
V_{av}	$\Delta R/R$	$\Delta V/V_a$	$\Delta\Theta(°)$	$\Delta R/R$	$\Delta V/V_a$	$\Delta\Theta(°)$	$\Delta R/R$	$\Delta V/V_a$	$\Delta\Theta(°)$
500	0.0021	0.0011	0.33	0.0053	0.0027	0.84	0.0095	0.0048	1.5
1000	0.0084	0.0084	1.3	0.021	0.021	3.2	0.041	0.041	5.0
1500	0.019	0.029	3.0	0.054	0.081	5.0	0.13	0.1	5.0
2000	0.035	0.070	5.0	0.13	0.1	5.0	0.26	0.1	5.0
2500	0.076	0.1	5.0	0.22	0.1	5.0	0.41	0.1	5.0
3000	0.12	0.1	5.0	0.32	0.1	5.0	0.5	0.1	5.0
				$\Theta = 60°$					
500	0.0040	0.0020	0.13	0.0099	0.0050	0.32	0.018	0.009	0.60
1000	0.016	0.016	0.053	0.040	0.040	1.3	0.071	0.071	2.3
1500	0.036	0.048	1.2	0.098	0.1	3.2	0.22	0.1	5.0
2000	0.069	0.1	2.3	0.22	0.1	5.0	0.47	0.1	5.0
2500	0.12	0.1	4.0	0.40	0.1	5.0	0.5	0.1	5.0
3000	0.17	0.1	5.0	0.5	0.1	5.0	0.5	0.1	5.0
				R = 3000' V_a = 500 ft./sec. $\Theta = 20°$					
500	0.0042	0.0021	0.65	0.011	0.006	1.7	0.019	0.010	3.0
1000	0.017	0.017	2.7	0.046	0.046	5.0	0.090	0.090	5.0
1500	0.041	0.062	5.0	0.15	0.1	5.0	0.28	0.1	5.0
2000	0.10	0.1	5.0	0.29	0.1	5.0	0.5	0.1	5.0
2500	0.18	0.1	5.0	0.45	0.1	5.0	0.5	0.1	5.0
3000	0.26	0.1	5.0	0.5	0.1	5.0	0.5	0.1	5.0
				$\Theta = 60°$					
500	0.0079	0.0040	0.26	0.020	0.010	0.66	0.036	0.018	1.2
1000	0.031	0.031	1.0	0.079	0.079	2.6	0.16	0.1	5.0
1500	0.073	0.1	2.4	0.30	0.1	5.0	0.5	0.1	5.0
2000	0.15	0.1	5.0	0.5	0.1	5.0	0.5	0.1	5.0
2500	0.30	0.1	5.0	0.5	0.1	5.0	0.5	0.1	5.0
3000	0.47	0.1	5.0	0.5	0.1	5.0	0.5	0.1	5.0

* Below $\frac{\Delta R}{R} = 0.5$, $\frac{\Delta V_a}{V_a} = 0.1$, $\Delta\Theta = 5.0$



$R = V_{AV}\, t =$ SLANT RANGE (ft) (V_{AV} DEFINED AS EQUAL TO $\left(\frac{R}{t}\right)$)

$M = \dfrac{g t^2 \cos\theta}{2} = \dfrac{g R^2 \cos\theta}{2 V_{AV}^2} =$ TRAJECTORY DROP (ft)

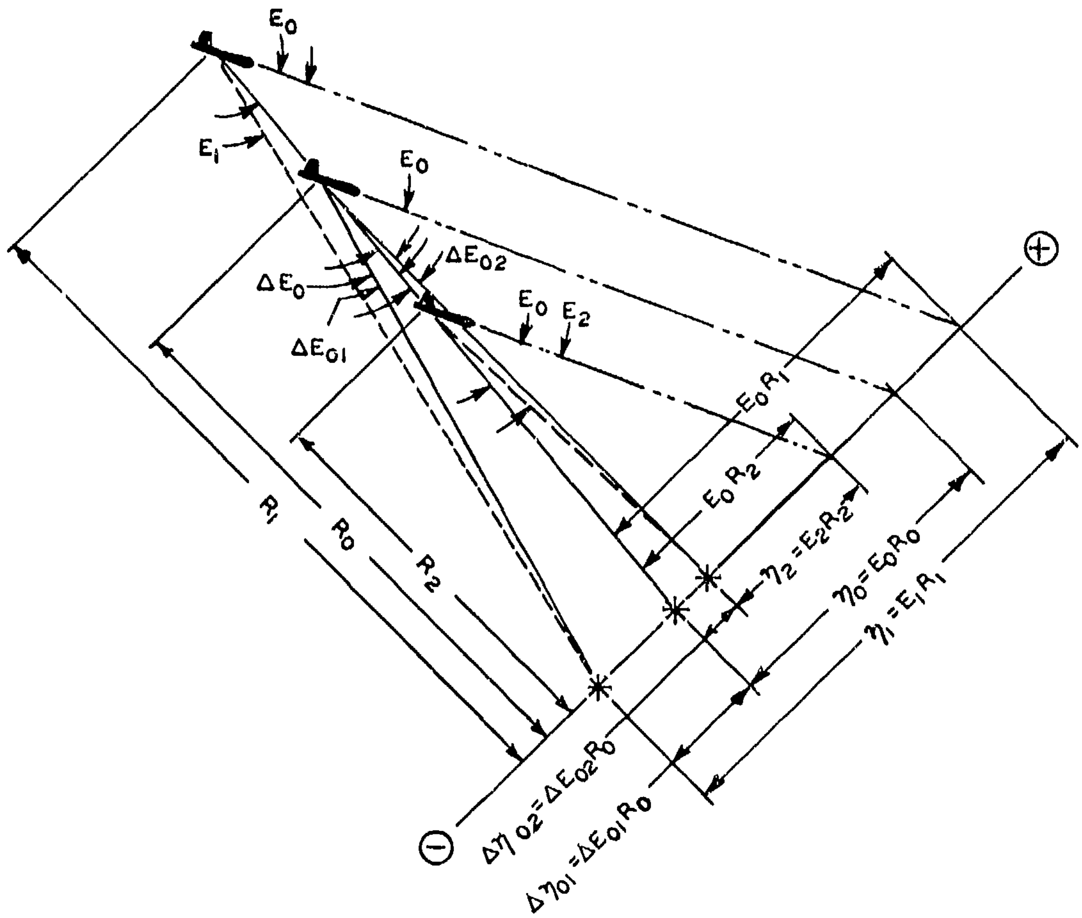
E (MILS) $= \dfrac{M}{R} = \dfrac{g R \cos\theta}{2 V_{AV}^2} =$ TRAJECTORY DROP (MILS)

I = TIME OF FLIGHT FROM RELEASE POINT TO GROUND (SEC.)

θ = ANGLE OF RELEASE

V_0 = SPEED OF RELEASE (FPS)

FIGURE 1. EFFECT OF GRAVITY UPON TRAJECTORY OF A PROJECTILE IN A VACUUM



ASSUME SIGHT SETS DEFLECTION ANGLE E_0 AT R_0 BUT RANGE INFORMATION IS UNCERTAIN WITHIN $R_1 = R_0 + \Delta R > R_0 > R_0 - \Delta R = R_2$

TOTAL DISPERSION AT TARGET (MIL) $= \Delta E_0 = \Delta E_{02} - \Delta E_{01} = \dfrac{(E_0R_2 - \eta_2)}{R_0} - \dfrac{(E_0R_1 - \eta_1)}{R_0}$

$$\doteq \frac{E_0R_2 - E_2R_2 - E_0R_1 + E_1R_1}{R_0} = \frac{R_2(E_0 - E_2) + R_1(E_1 - E_0)}{R_0}$$

$$\doteq \frac{(R_0 - \Delta R)(E_0 - E_2) + (R_0 + \Delta R)(E_1 - E_0)}{R_0}$$

$$\doteq E_1 - E_2 + \frac{\Delta R}{R_0}\left[(E_1 - E_0) - (E_0 - E_2)\right]$$

BUT $(E_1 - E_0) \doteq \text{CONST.} \times \Delta R \doteq (E_0 - E_2)$

SO, $\Delta E_0 = E_1 - E_2$

THE LOCATION OF THE MEAN CENTER OF IMPACT (MIL) $\doteq - \text{COUST}\left(\dfrac{\Delta R^2}{R_0}\right)$

ASSUME THIS CORRECTION IS INSERTED IN E_0, KNOWING RANGE OF $\dfrac{\Delta R}{R_0}$

THEN INCREMENT OF LINEAR ERROR FROM MEAN (MIL) $= \pm \dfrac{E_1 - E_2}{2}$

ASSUME $\sigma_R = \dfrac{E_1 - E_2}{2} \doteq \text{CONST.}\ (\Delta R)$

FIGURE 2. ASSUMPTIONS FOR RANGE ERROR

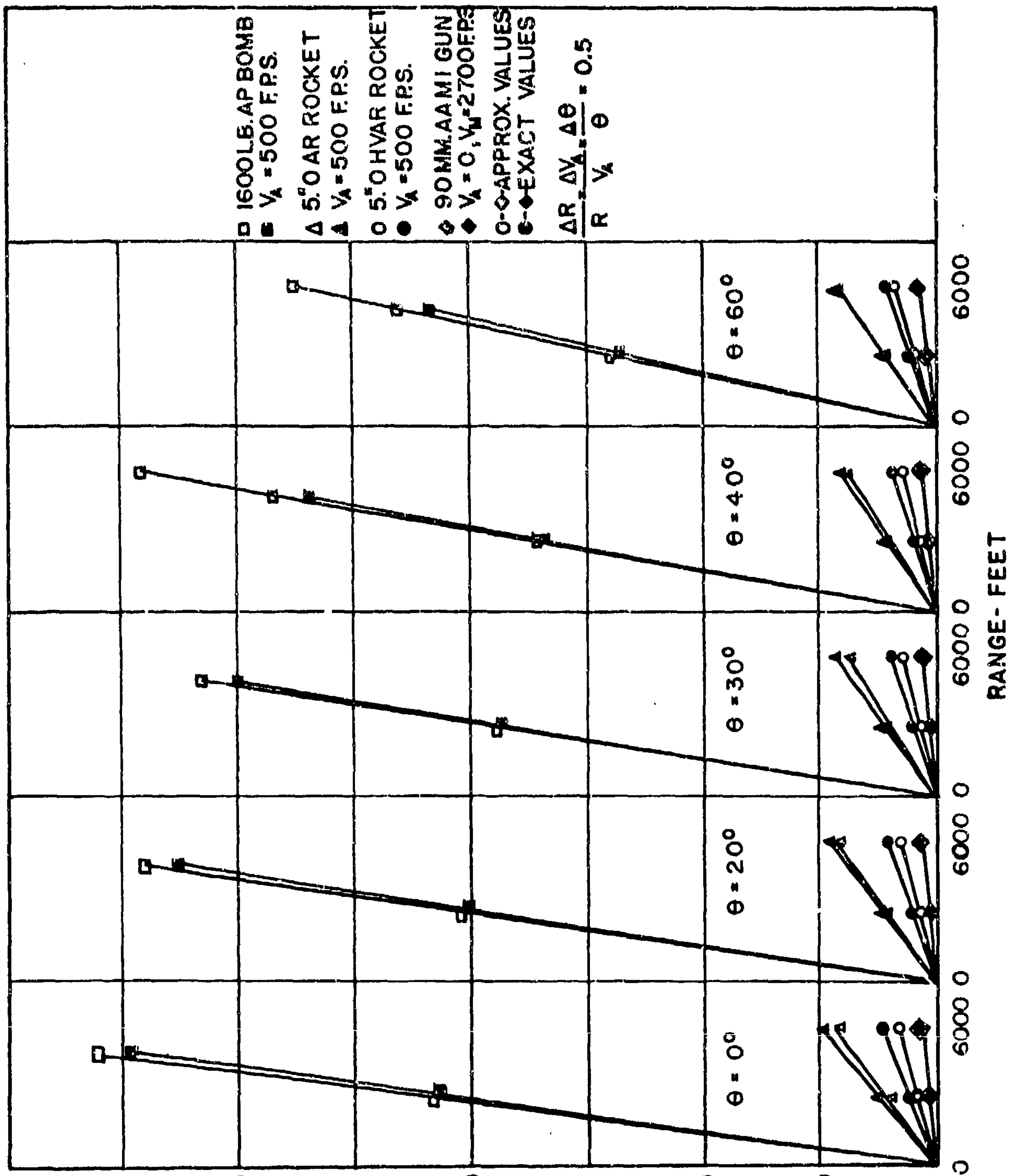


FIGURE 3. COMPARISON OF APPROXIMATE AND EXACT $\Delta \Sigma g$'s

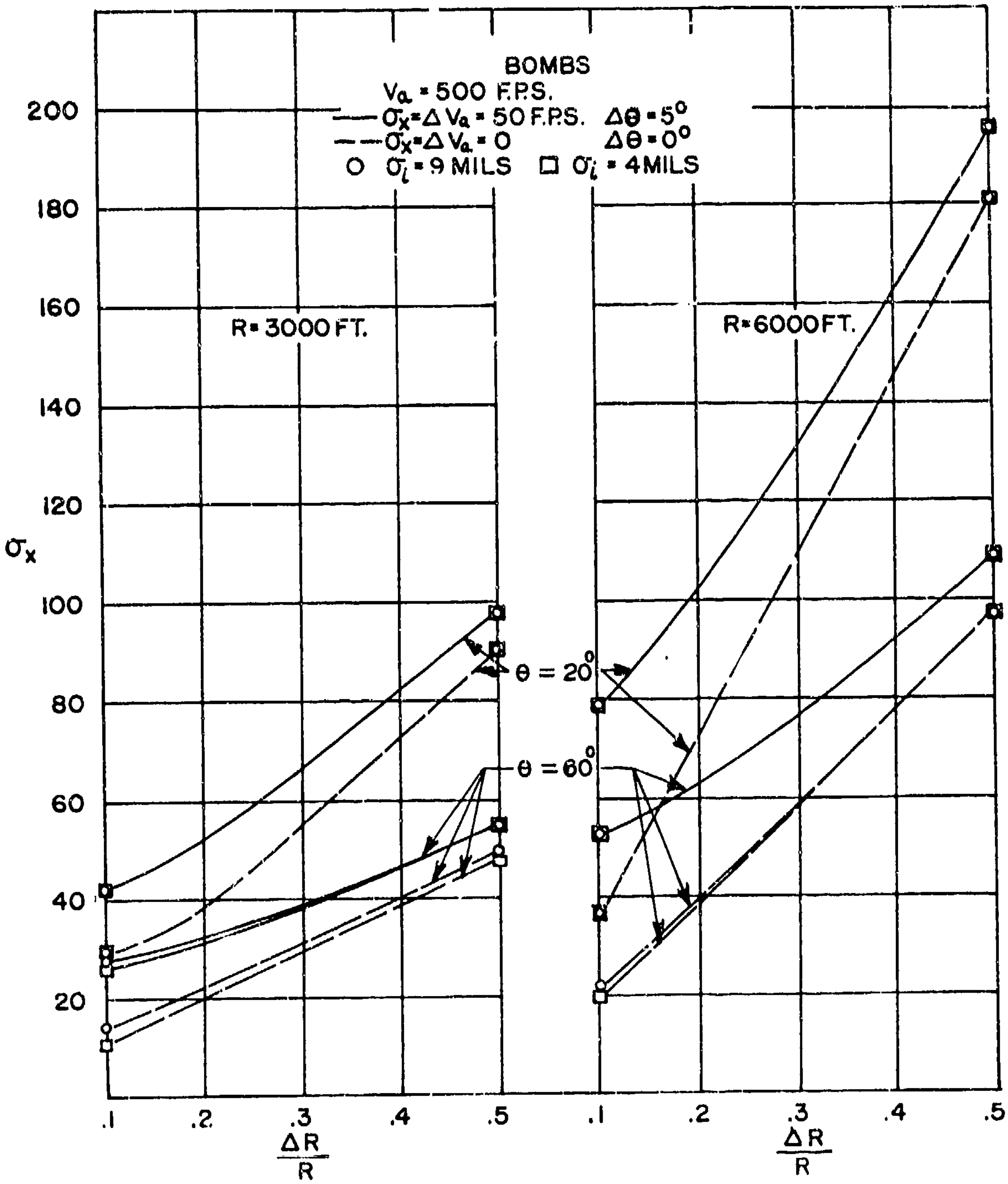


FIGURE 4 A. EFFECT OF UNCERTAINTIES AS TO POINT OF RELEASE ON DISPERSION OF BOMBS

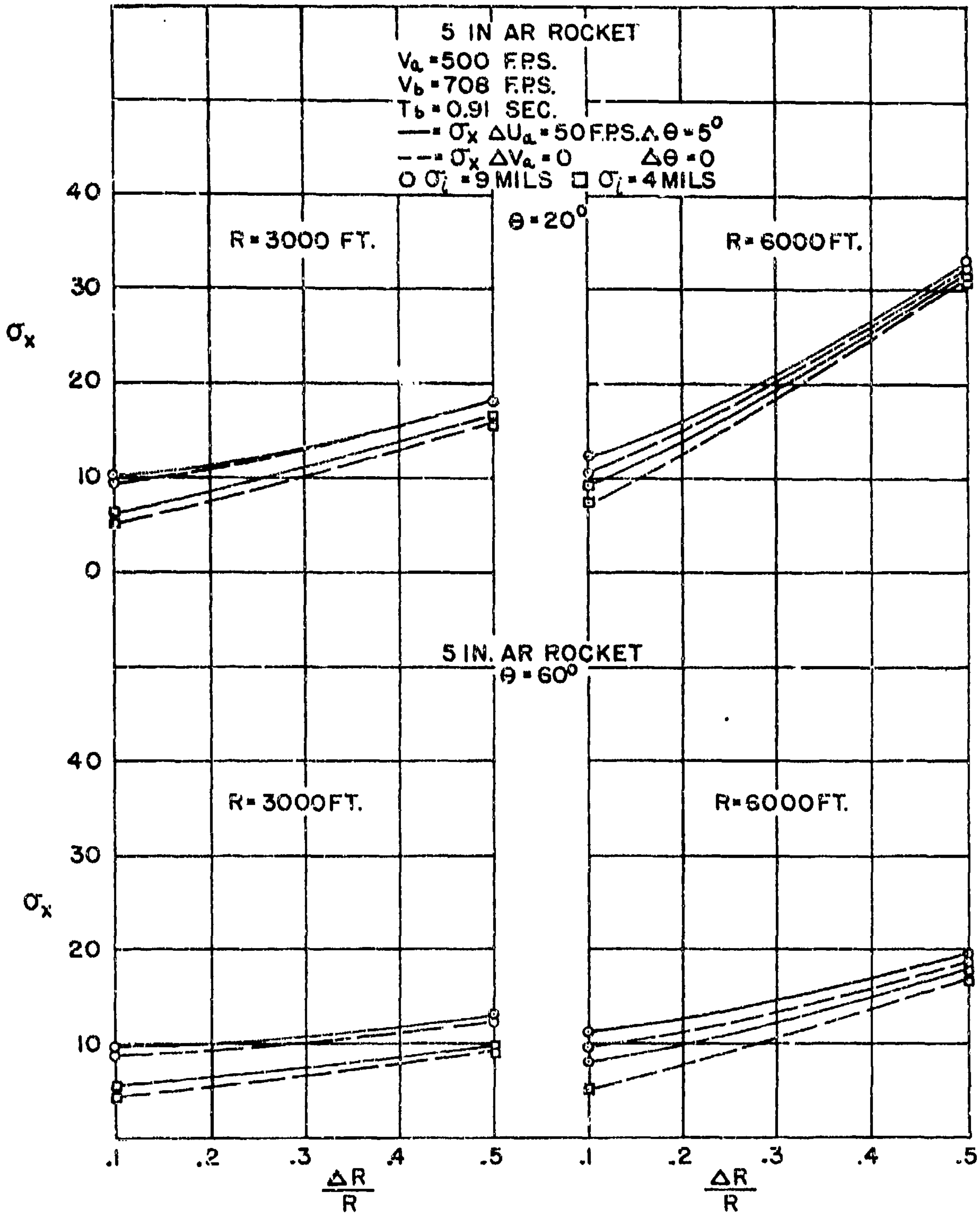


FIGURE 4B. EFFECT OF UNCERTAINTIES AS TO POINT OF RELEASE ON DISPERSION OF BOMBS

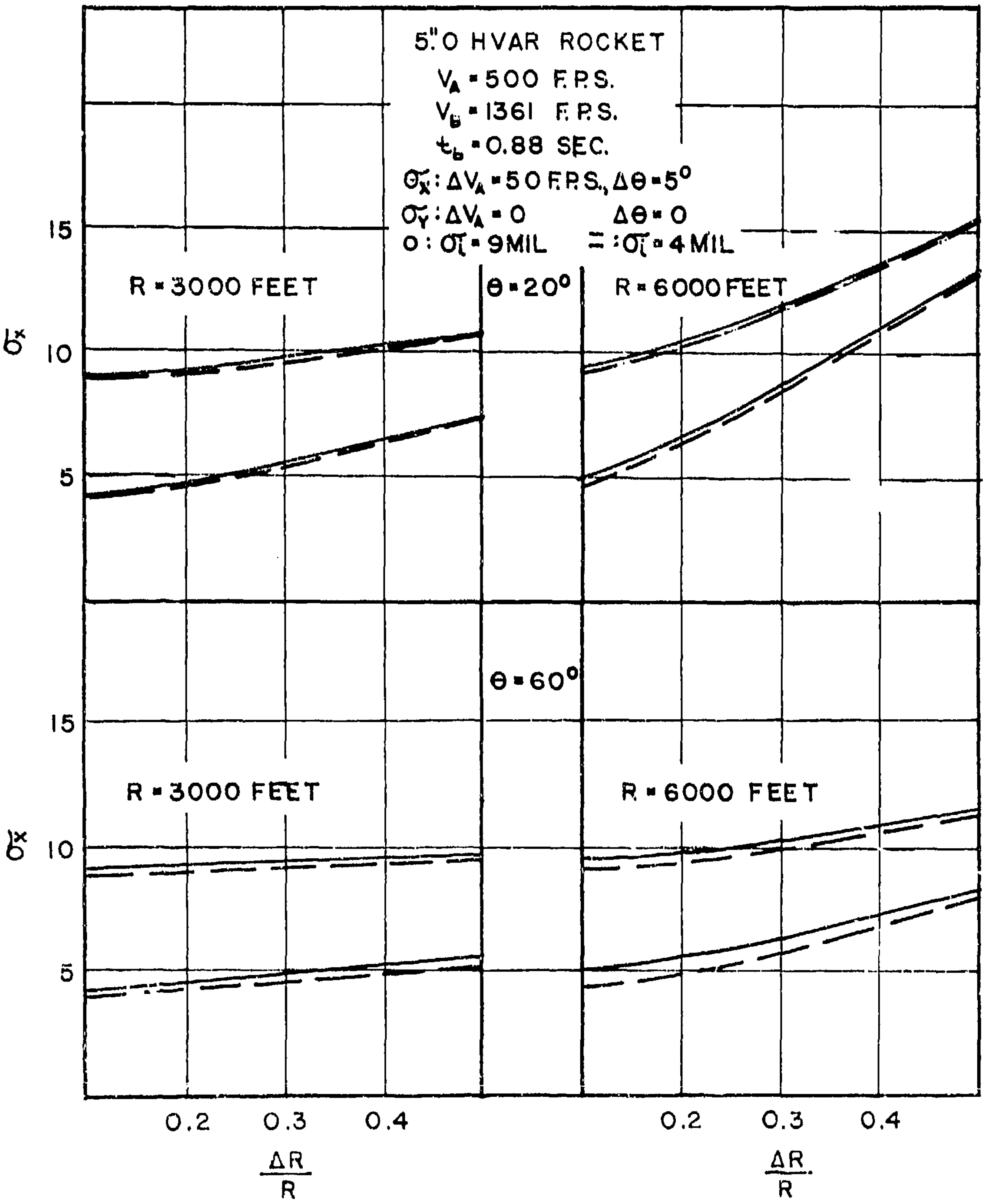


FIGURE 4C. EFFECT OF RELEASE ERROR ON σ_Y

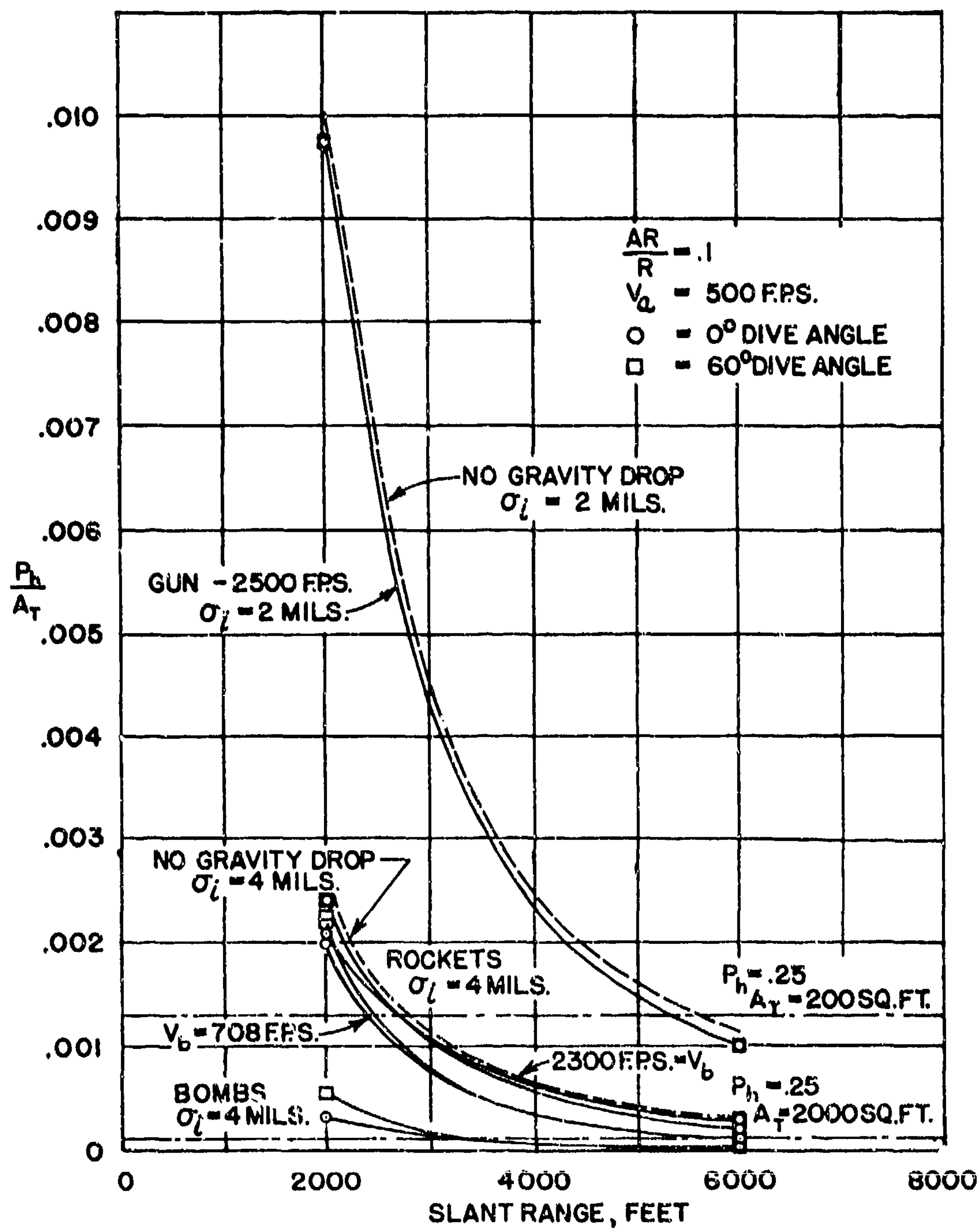


FIGURE 5 A. EFFECT OF RANGE UPON HIT PROBABILITY PER SQUARE FOOT OF TARGET AREA

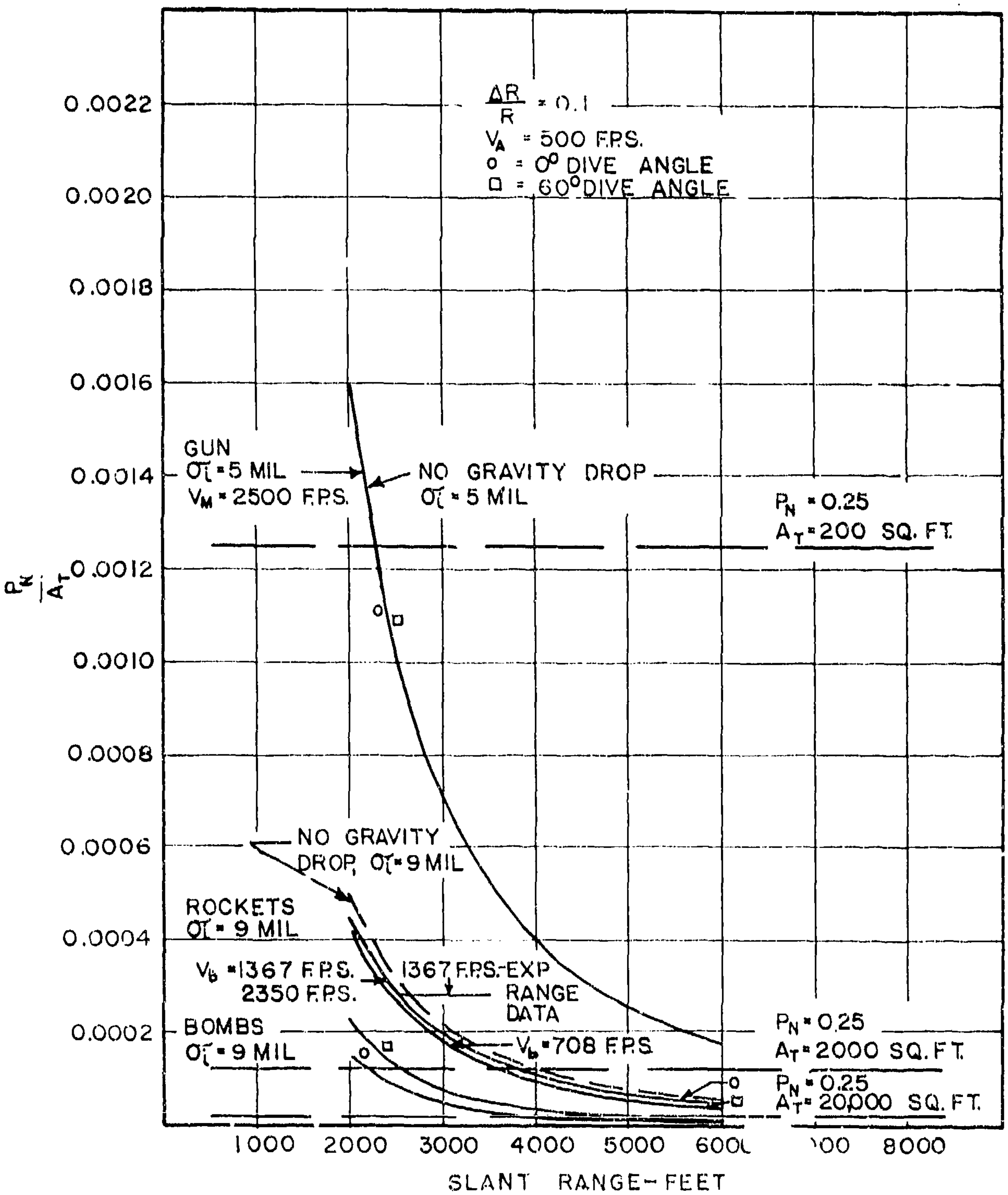


FIGURE 5B. PROBABILITY OF HIT PER SQUARE FOOT OF TARGET AREA AS A FUNCTION OF RANGE

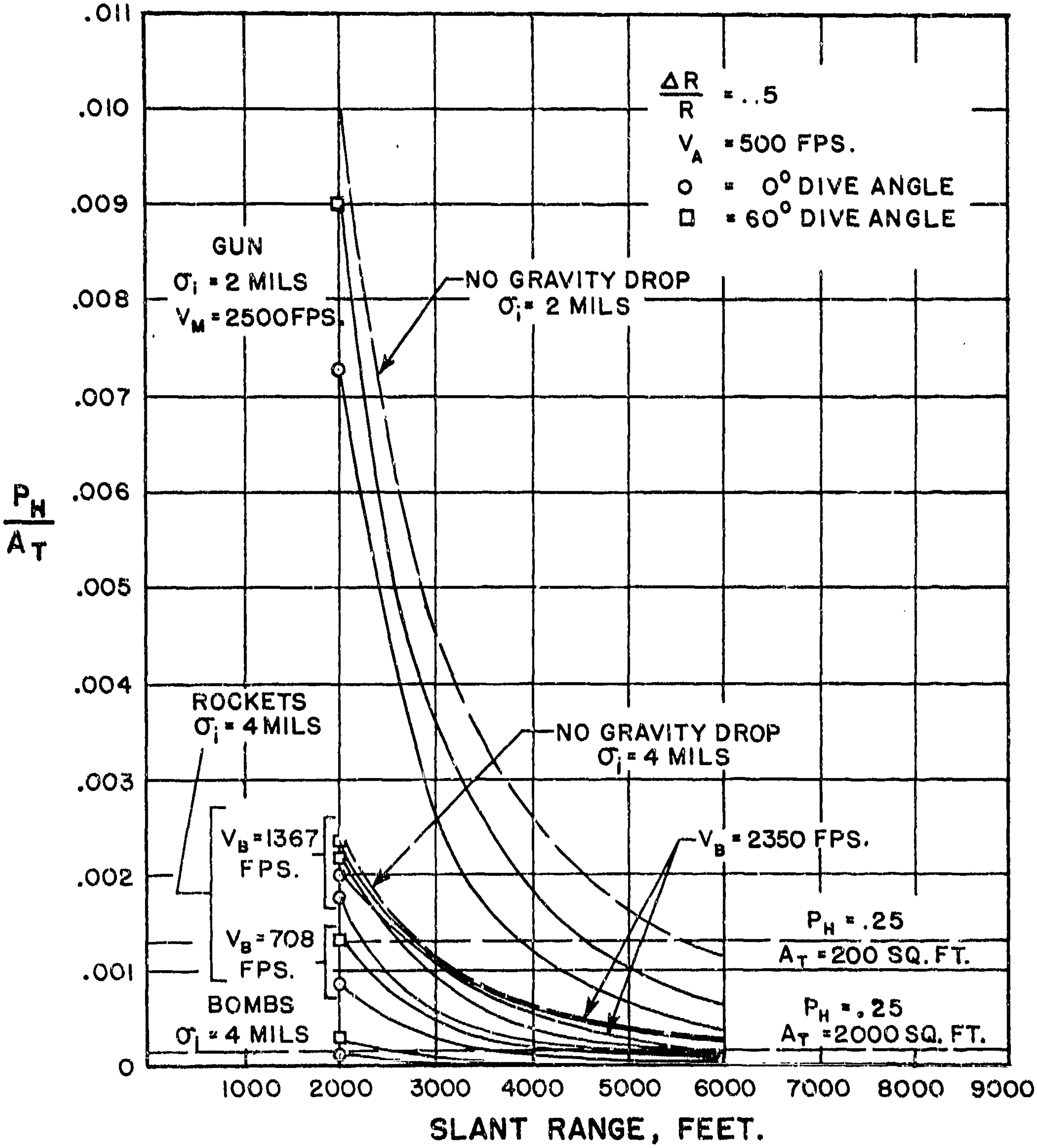


FIGURE 5c-EFFECT OF RANGE UPON HIT PROBABILITY PER SQUARE FOOT OF TARGET AREA.

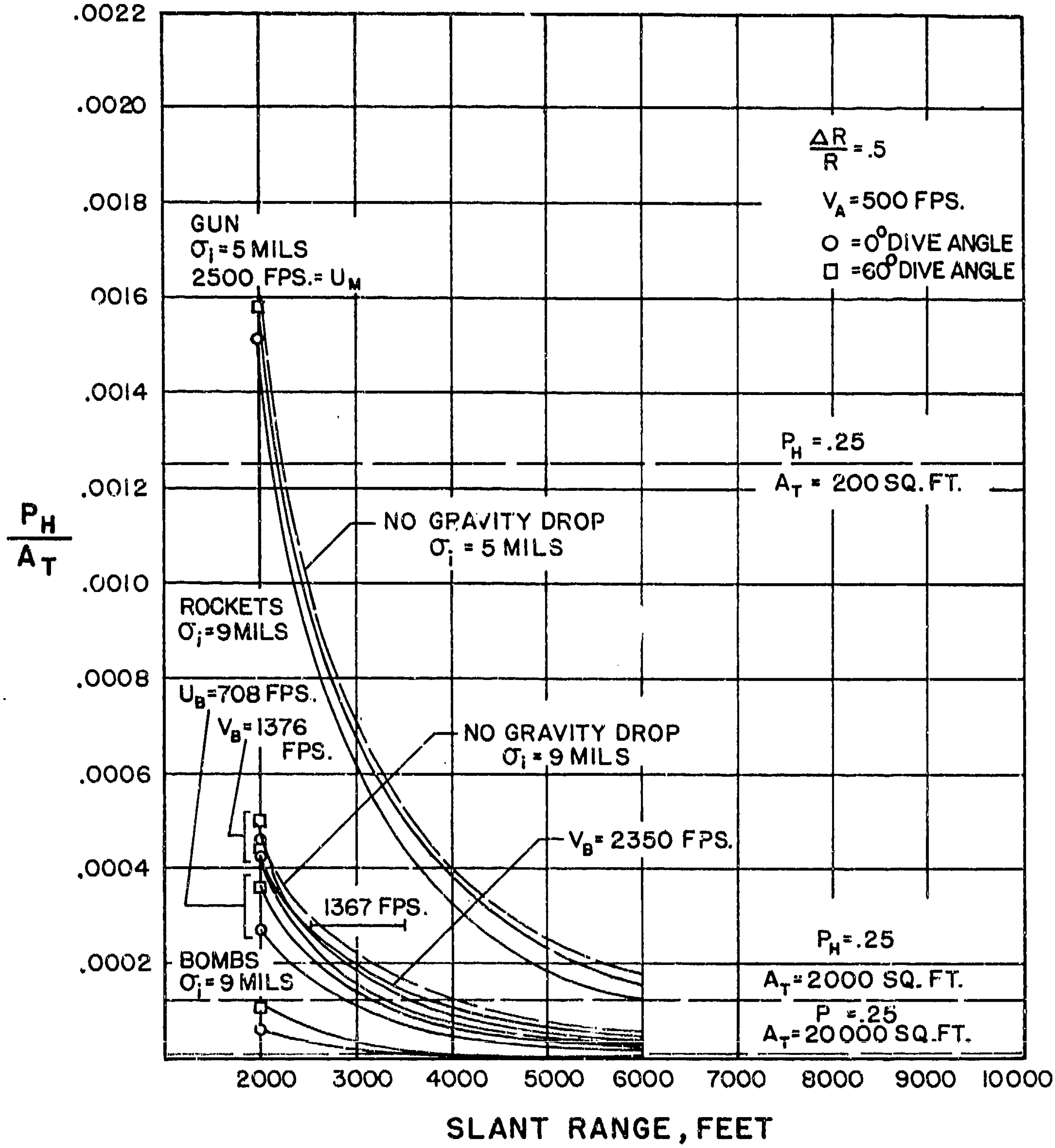


FIGURE 5a - EFFECT OF RANGE UPON HIT PROBABILITY PER SQUARE FOOT OF TARGET AREA.

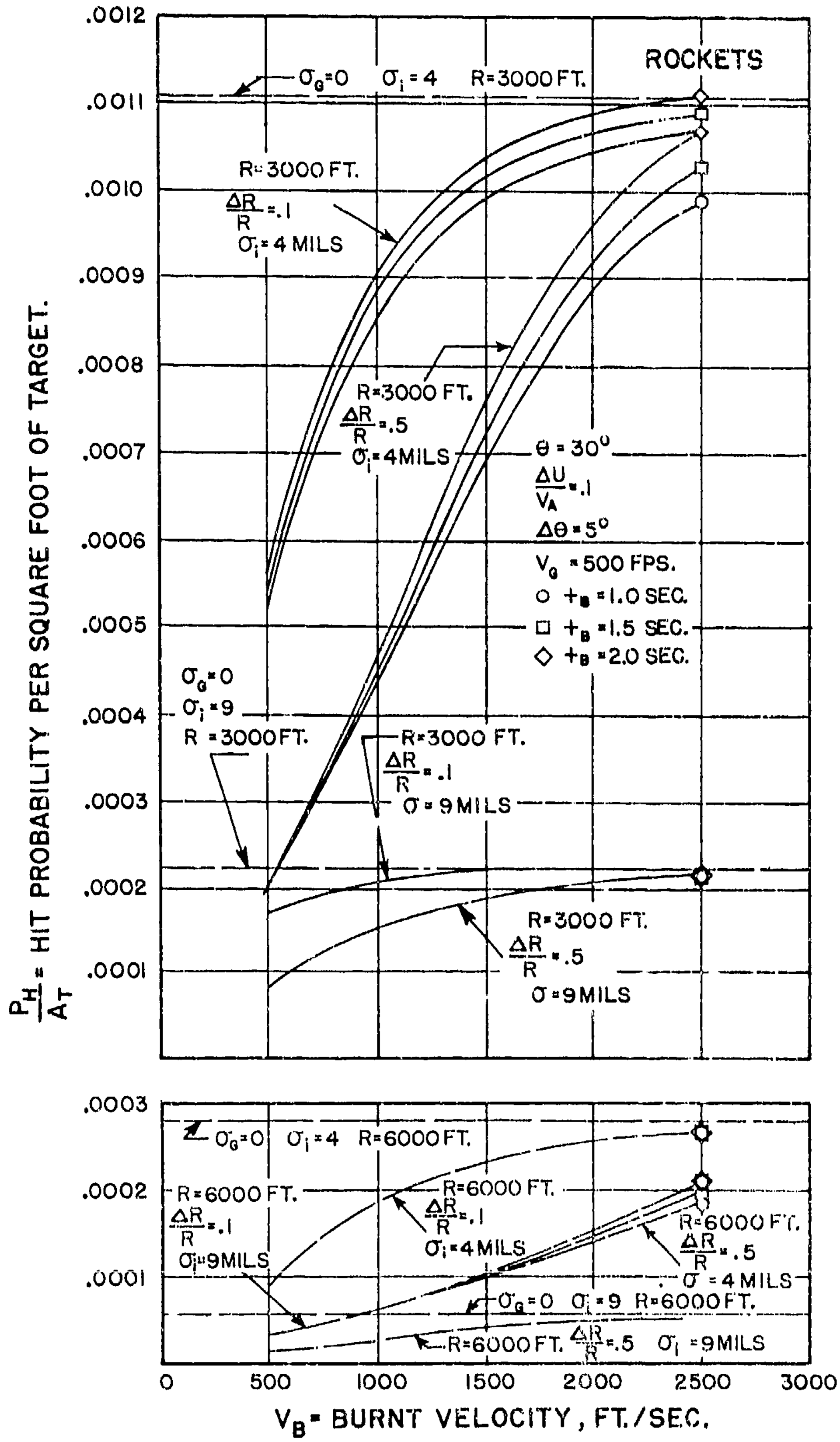


FIGURE 6a–EFFECT OF VELOCITY OF PROJECTILE RELATIVE TO AIRCRAFT UPON HIT PROBABILITY PER SQUARE FOOT OF TARGET.

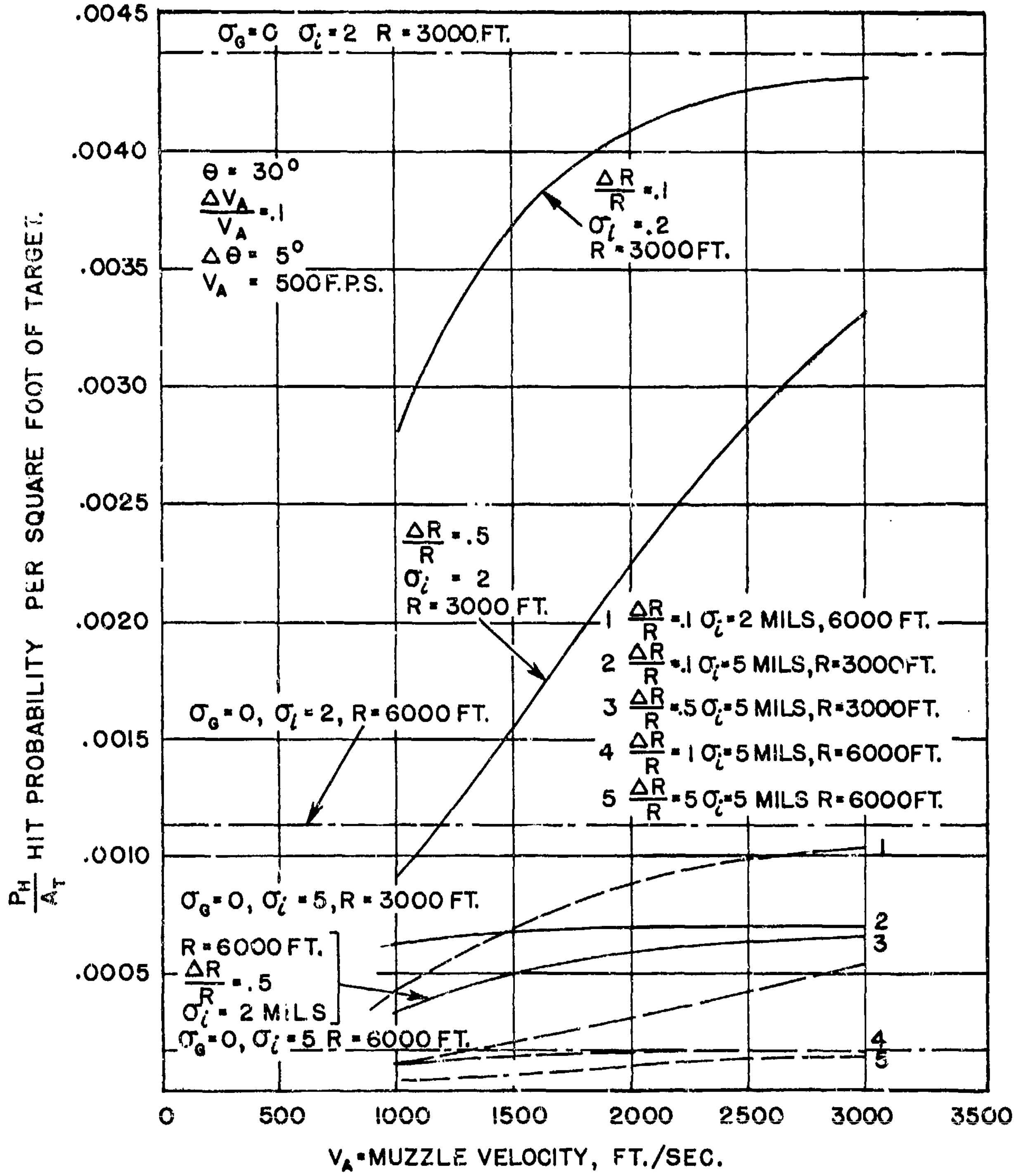


FIGURE 6 B. EFFECT OF VELOCITY OF PROJECTILE RELATIVE TO AIRCRAFT UPON HIT PROBABILITY PER SQUARE FOOT OF TARGET

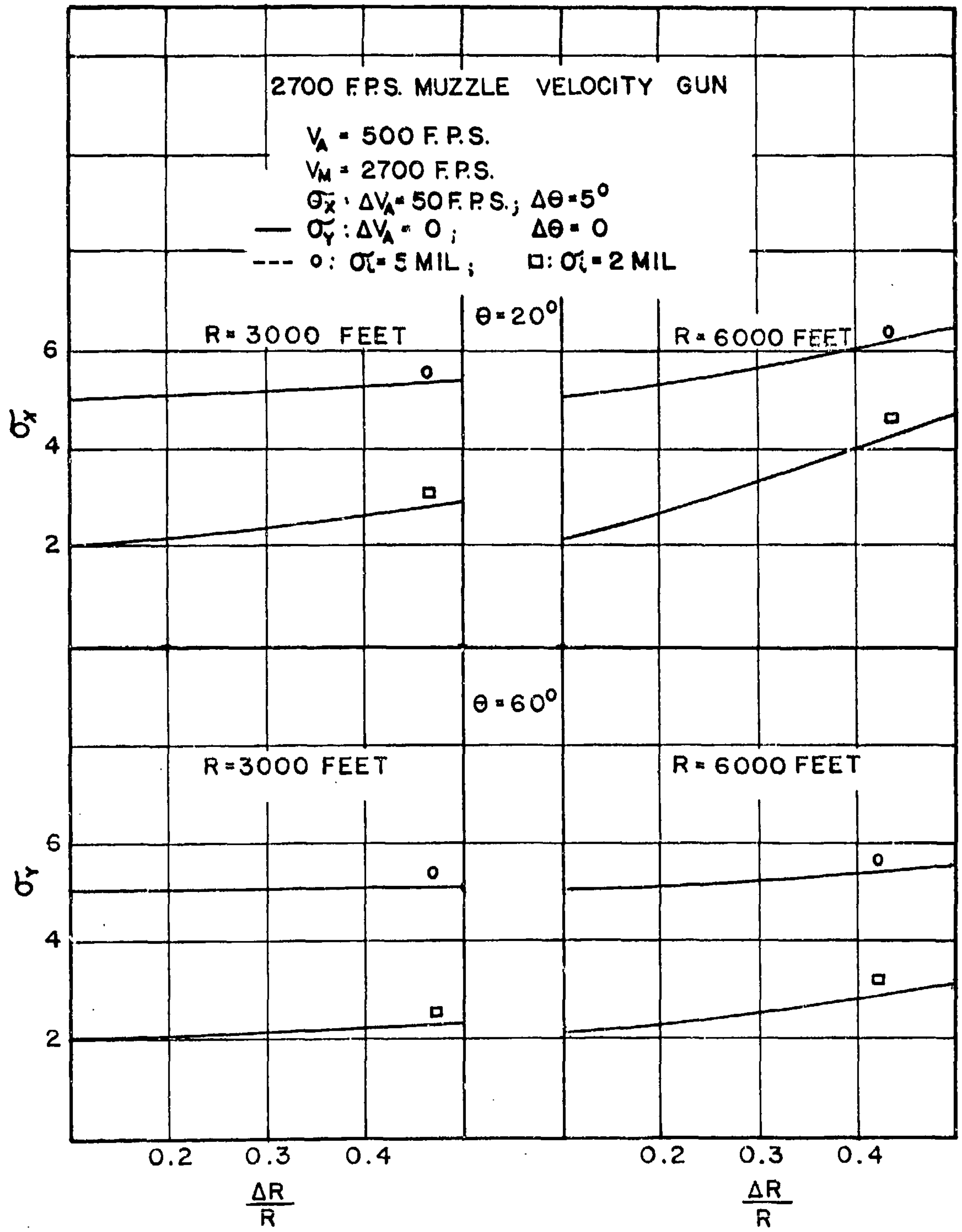


FIGURE 7. EFFECT OF RELEASE ERROR ON $\sigma_{\bar{X}}$

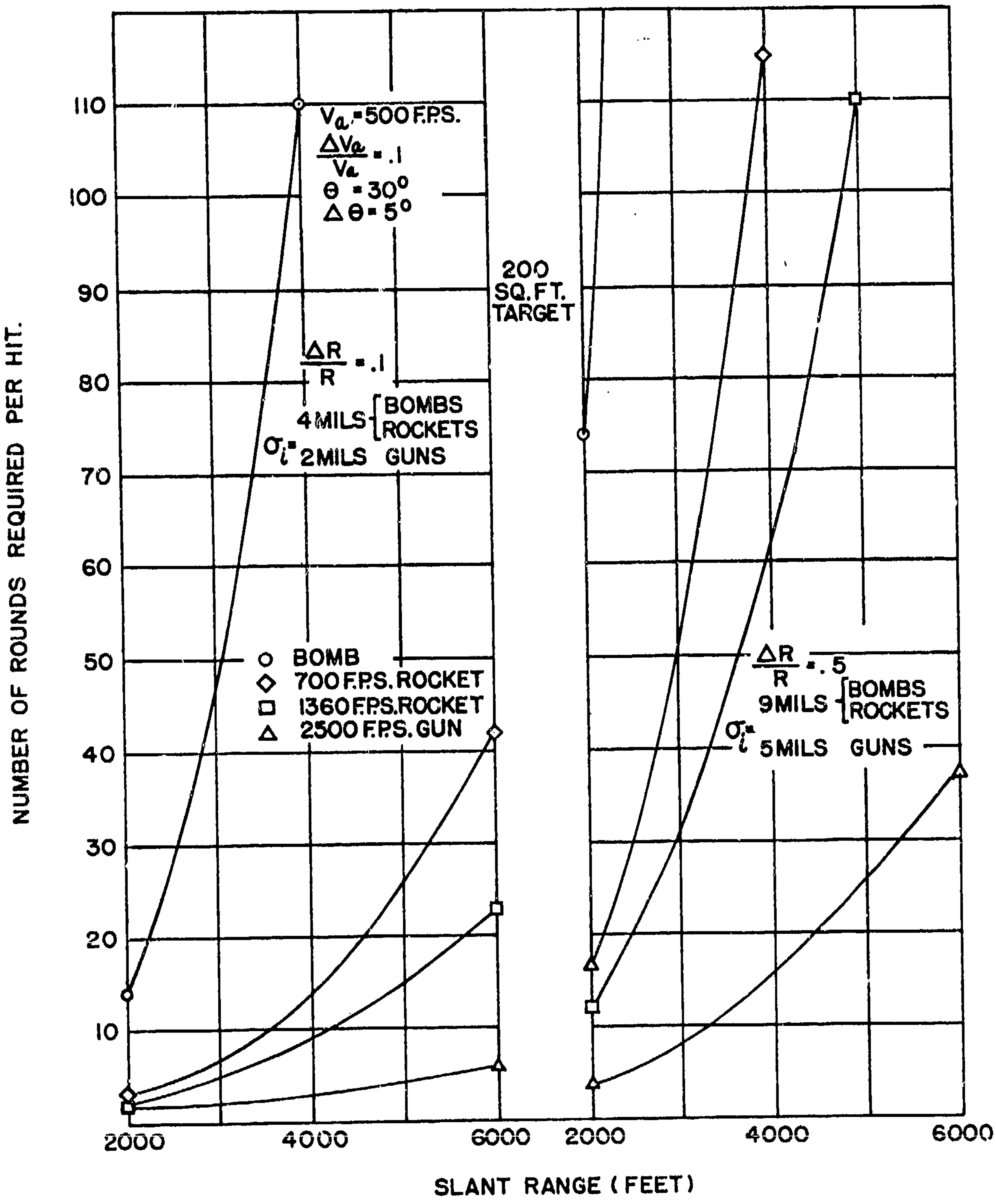


FIGURE 7A. EFFECT OF SLANT RANGE UPON ROUNDS REQUIRED PER HIT FOR A SPECIFIED TARGET AREA

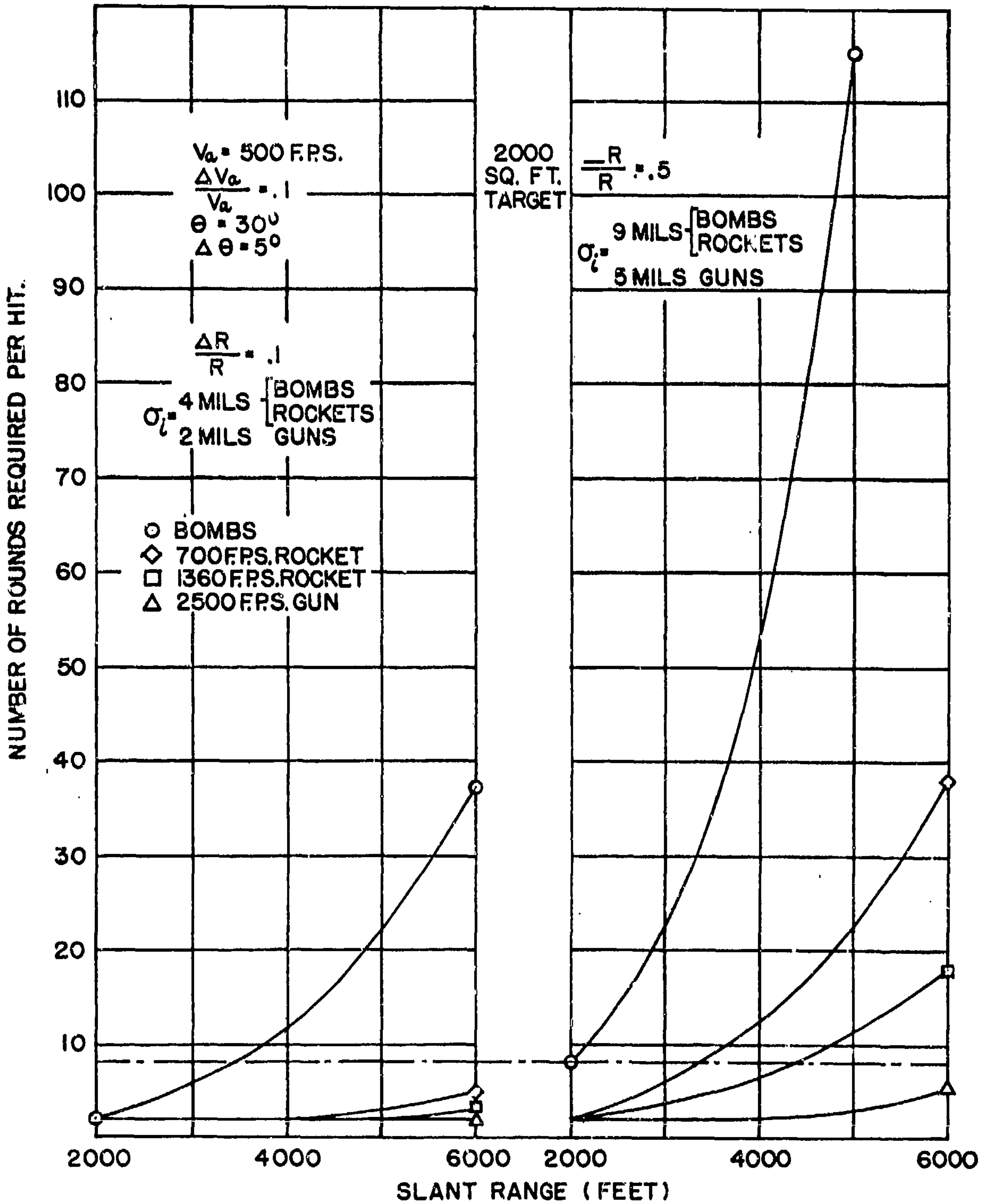


FIGURE 7 B. EFFECT OF SLANT RANGE UPON ROUNDS REQUIRED PER HIT FOR A SPECIFIED TARGET AREA

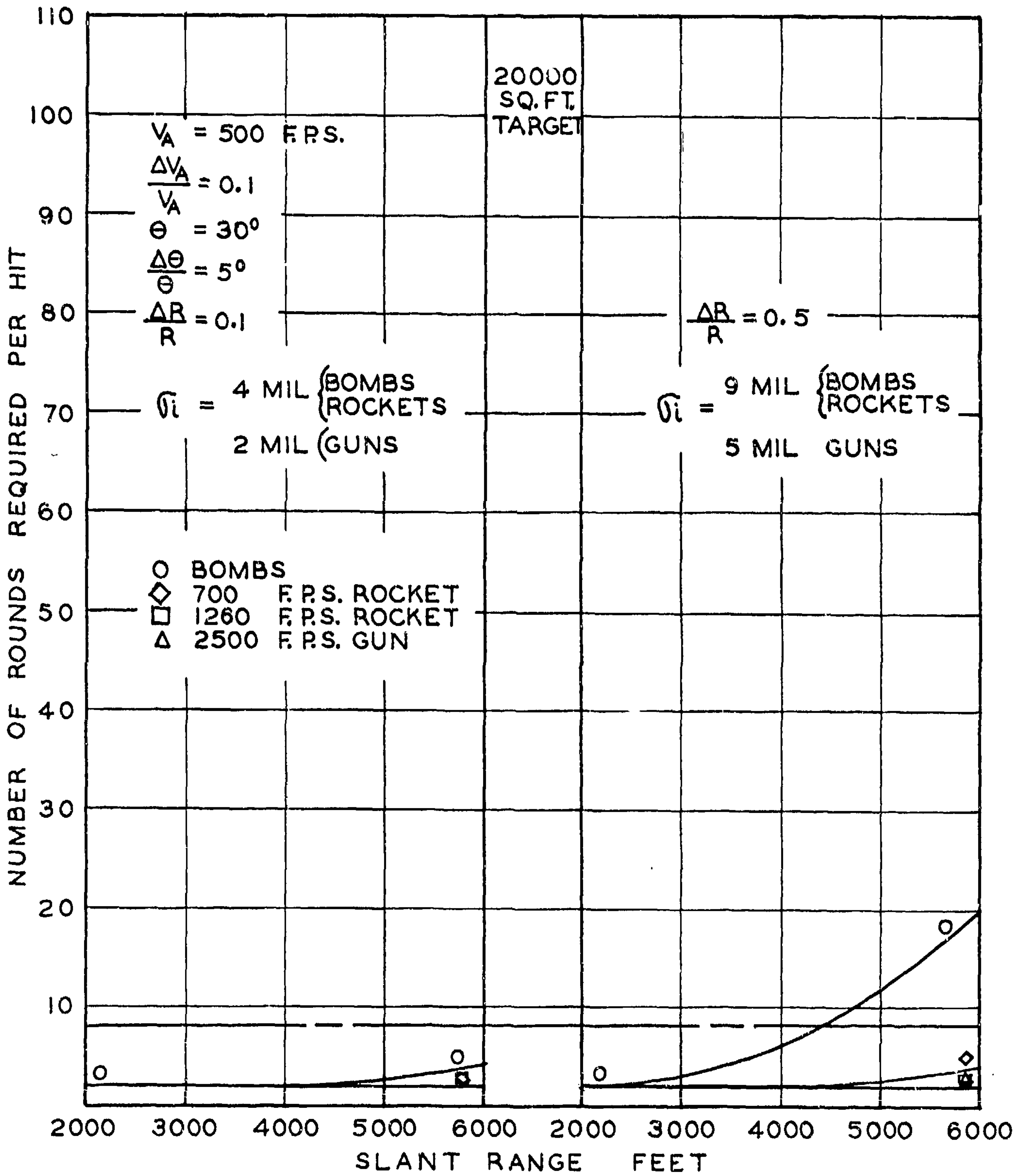


FIGURE 7C. NUMBER OF ROUNDS REQUIRED PER HIT AS A FUNCTION OF RANGE - TARGET AREA = 20,000 SQUARE FEET

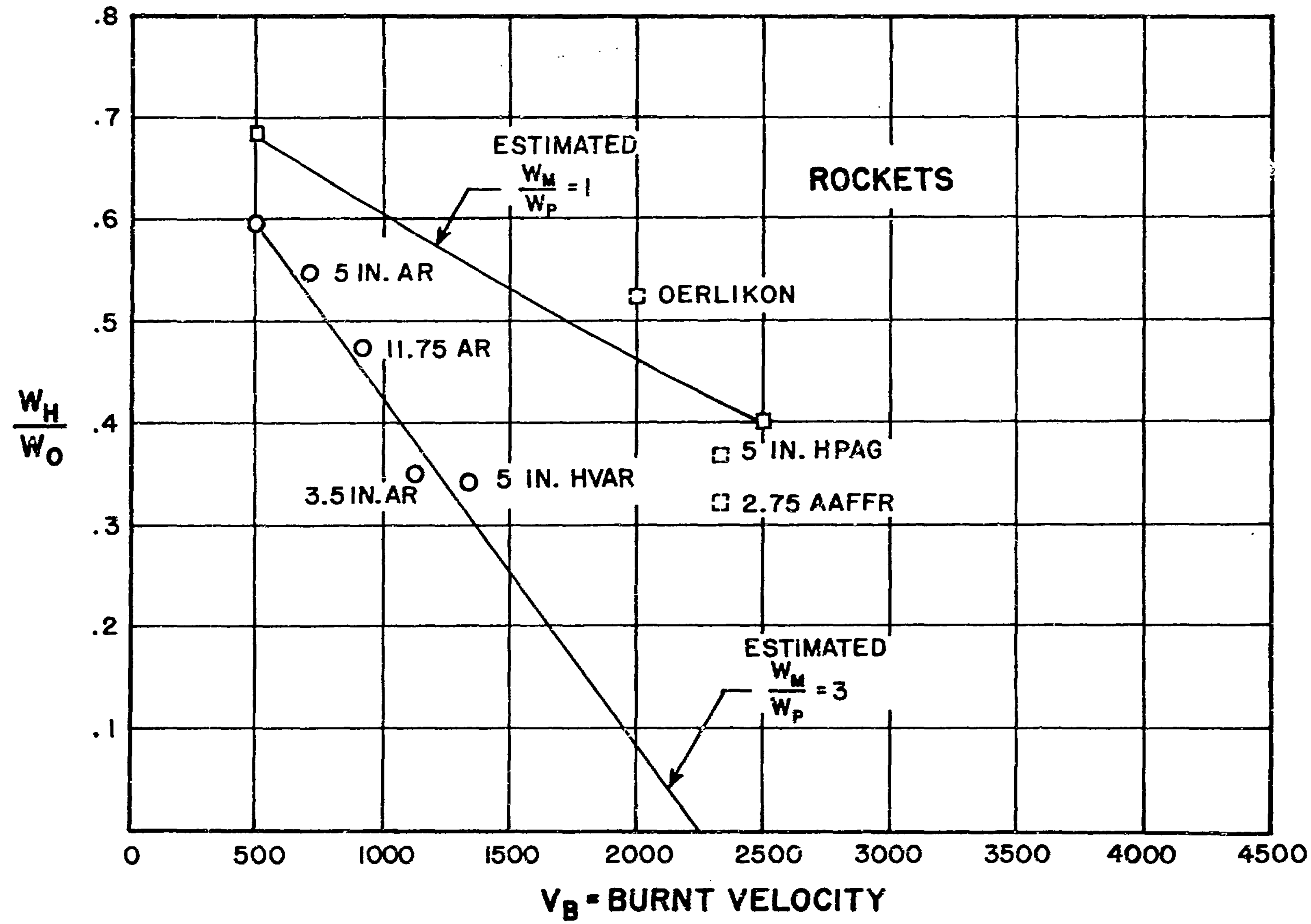


FIGURE 8—EFFECT OF BURNT VELOCITY OF ROCKET UPON RATIO OF WEIGHT OF WARHEAD TO THAT OF TOTAL ARMAMENT SYSTEM.

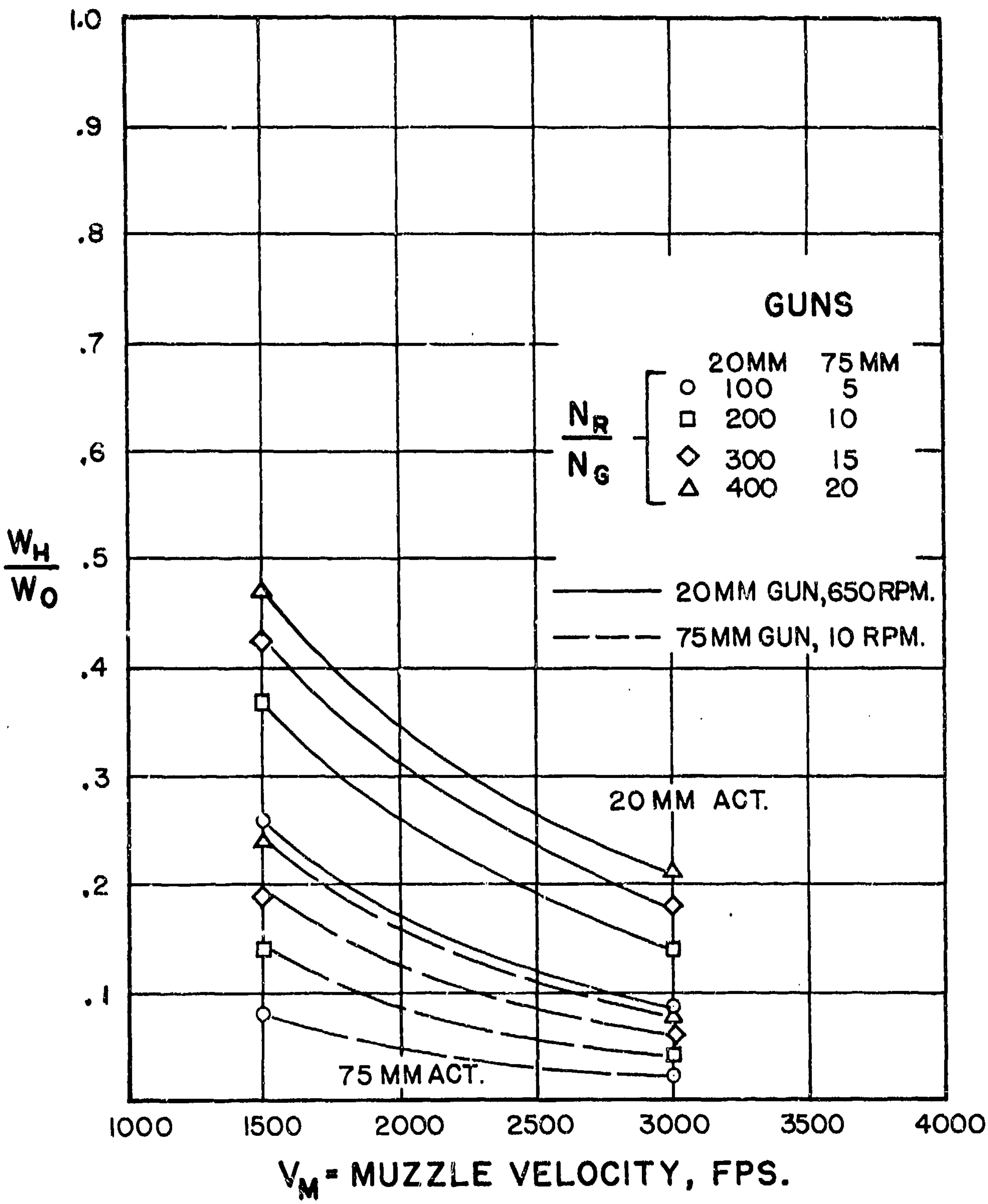


FIGURE 9 - EFFECT OF MUZZLE VELOCITY OF GUN PROJECTILE UPON RATIO OF WEIGHT OF WARHEAD TO THAT OF TOTAL ARMAMENT SYSTEM.

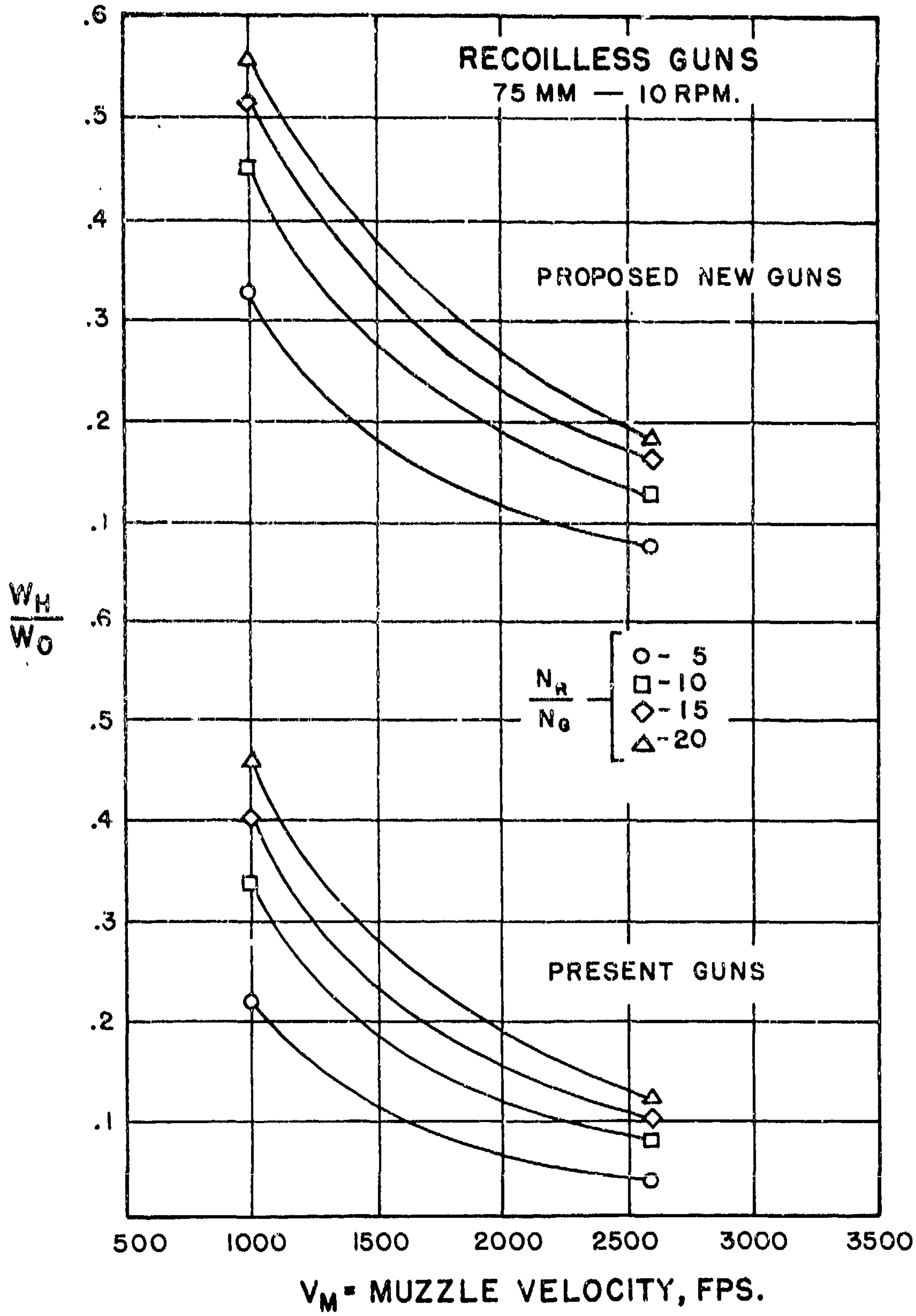


FIGURE 10 - EFFECT OF MUZZLE VELOCITY OF PROJECTILES FROM RECOILLESS GUNS UPON RATIO OF WEIGHT OF WARHEAD TO THAT OF TOTAL ARMAMENT SYSTEM.

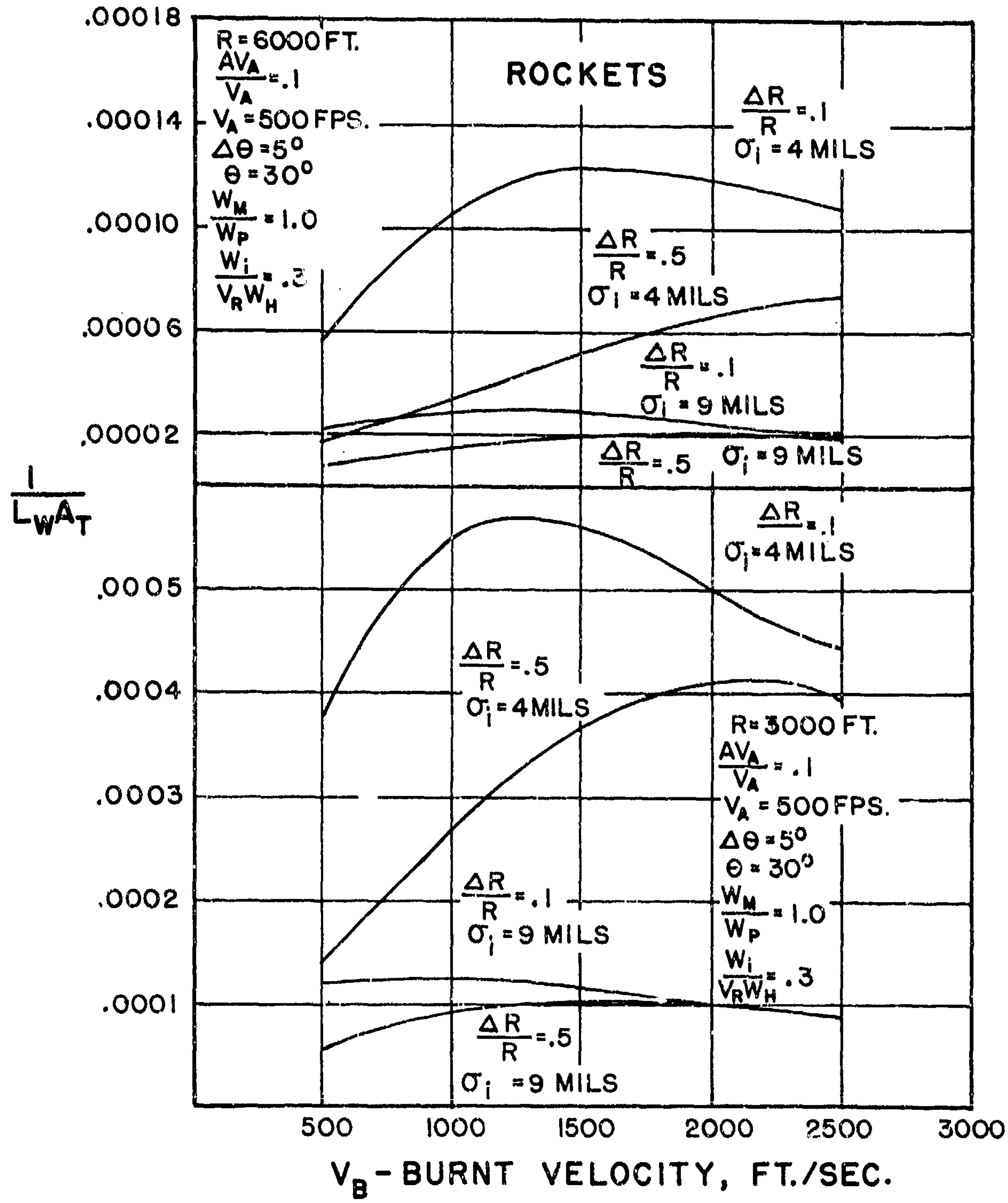


FIGURE 11a– RECIPROCAL OF AIRCRAFT ARMAMENT LOGISTIC FACTOR AS A FUNCTION OF ROCKET BURNT VELOCITY.

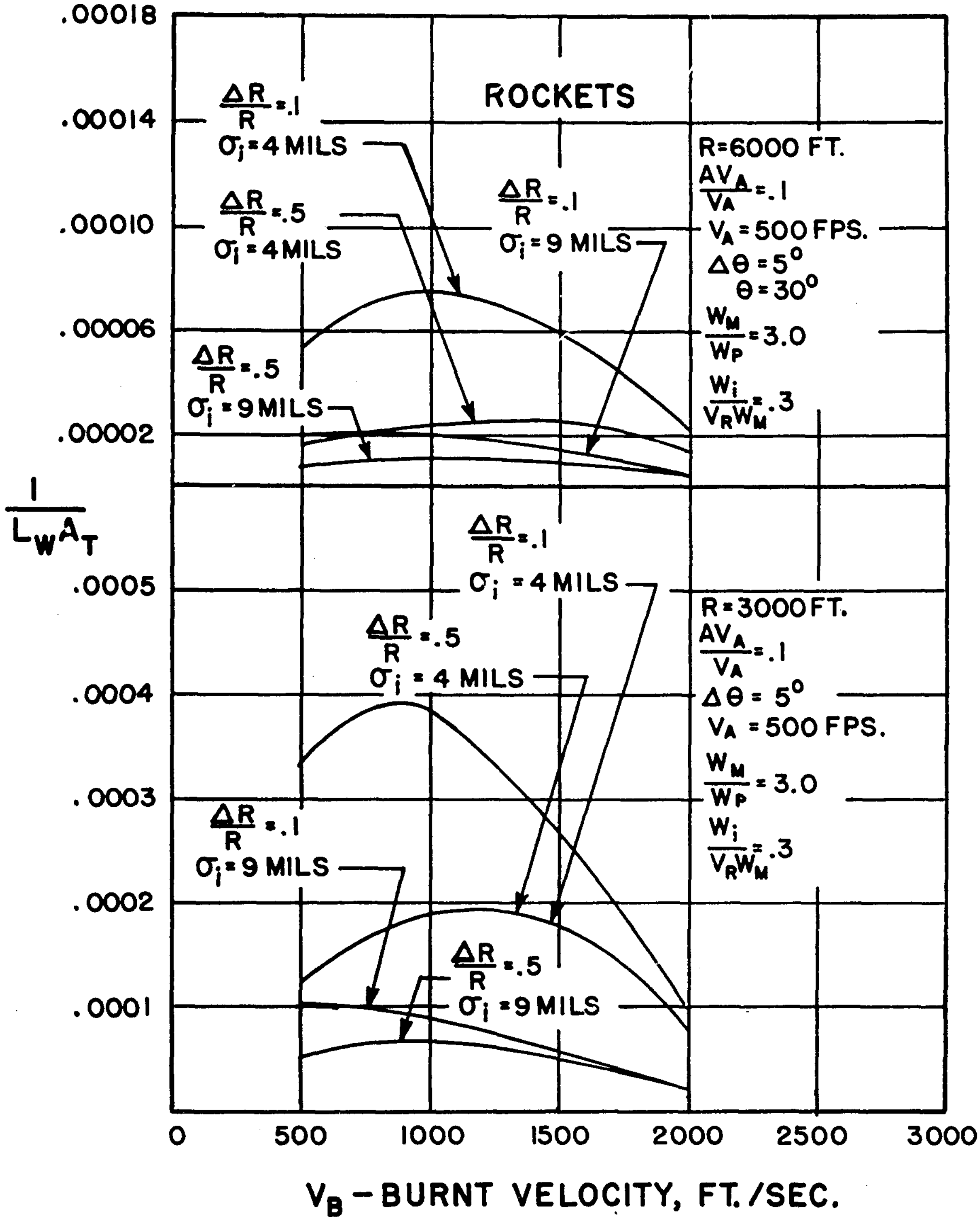


FIGURE 11b – RECIPROCAL OF AIRCRAFT ORDNANCE LOGISTIC FACTOR AS A FUNCTION OF ROCKET BURNT VELOCITY.

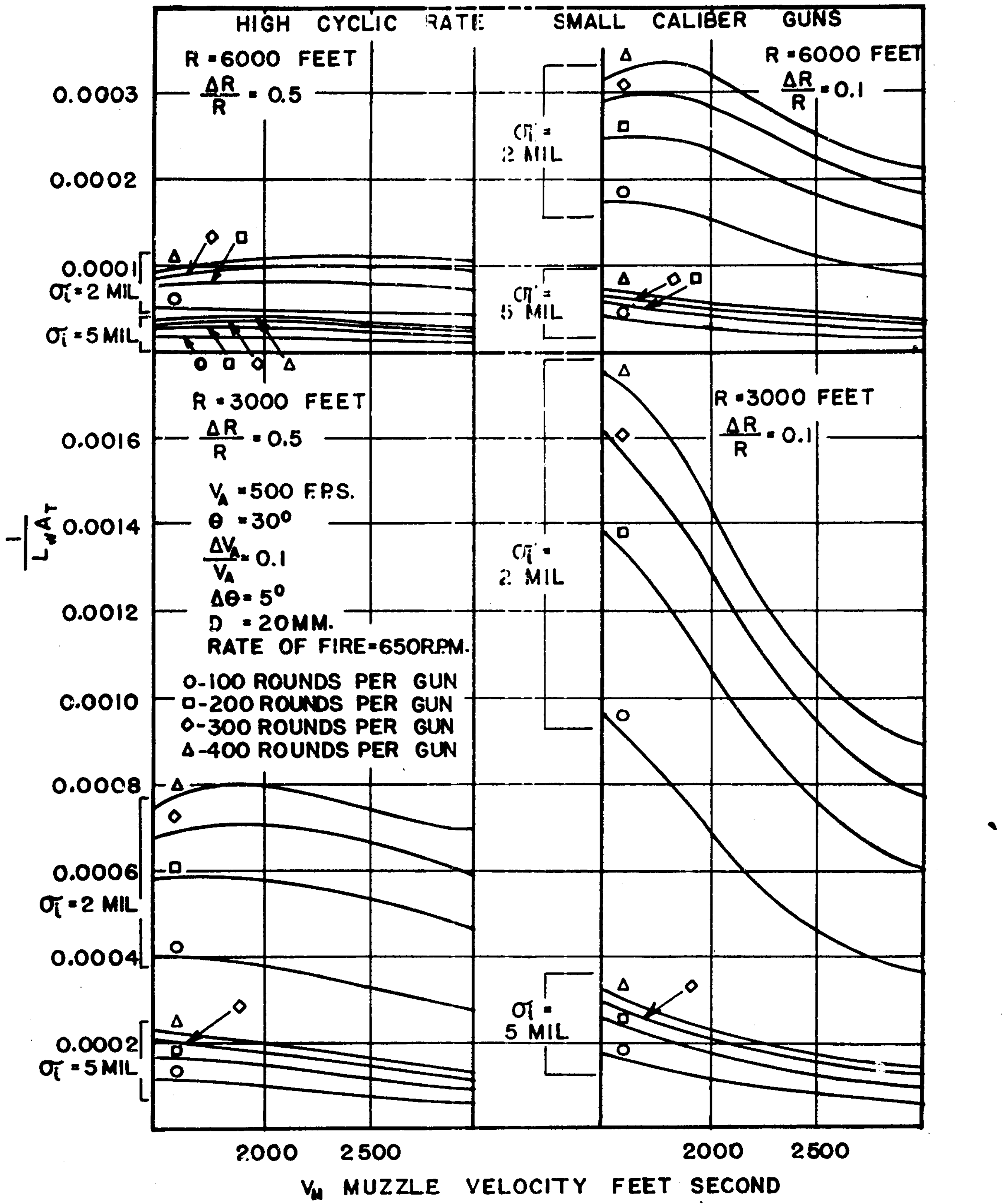


FIGURE 12A. RECIPROCAL OF AIRCRAFT ARMAMENT LOGISTIC FACTOR AS A FUNCTION OF GUN MUZZLE VELOCITY

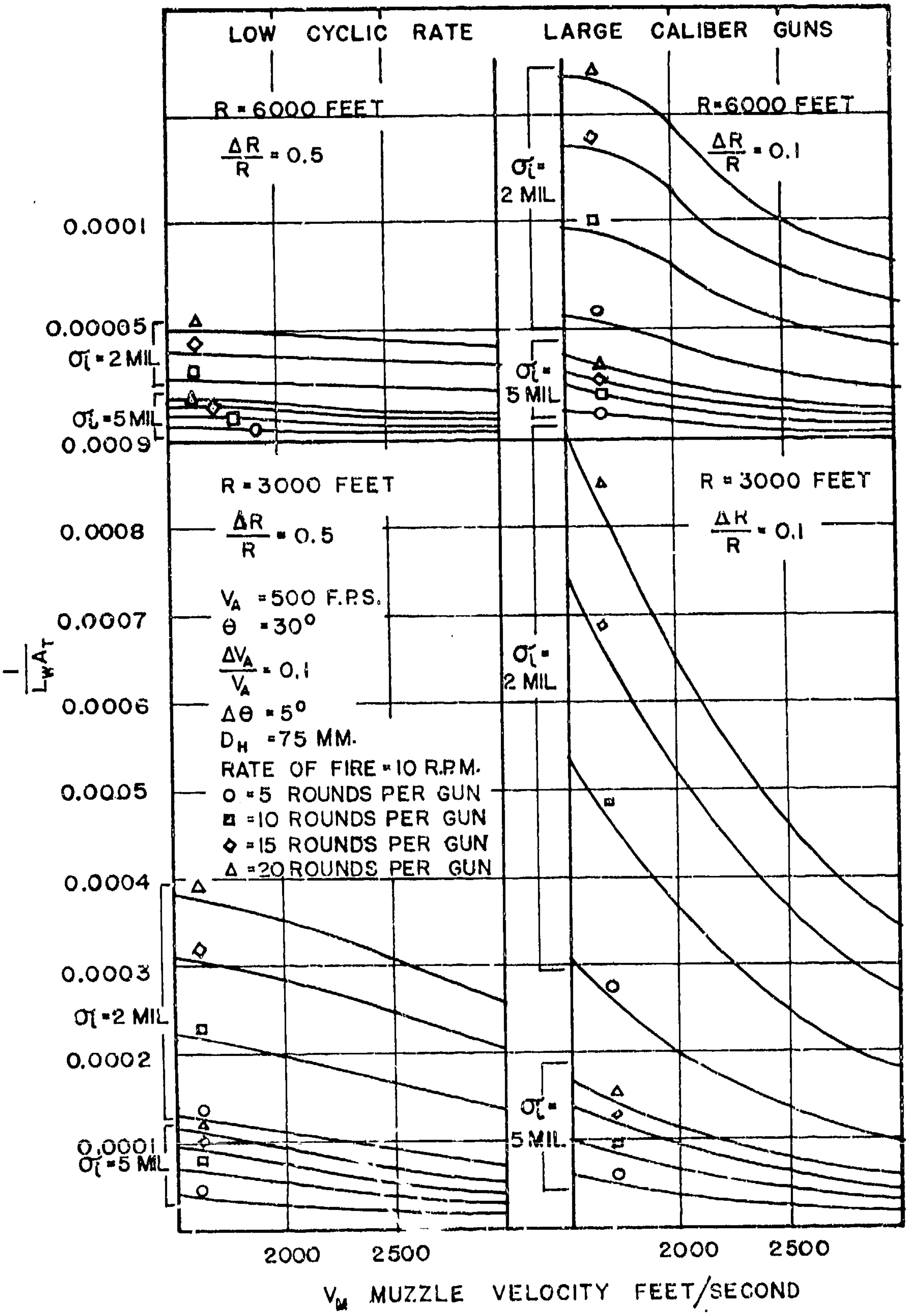


FIGURE 12B. RECIPROCAL OF AIRCRAFT ARMAMENT LOGISTIC FACTOR AS A FUNCTION OF GUN MUZZLE VELOCITY

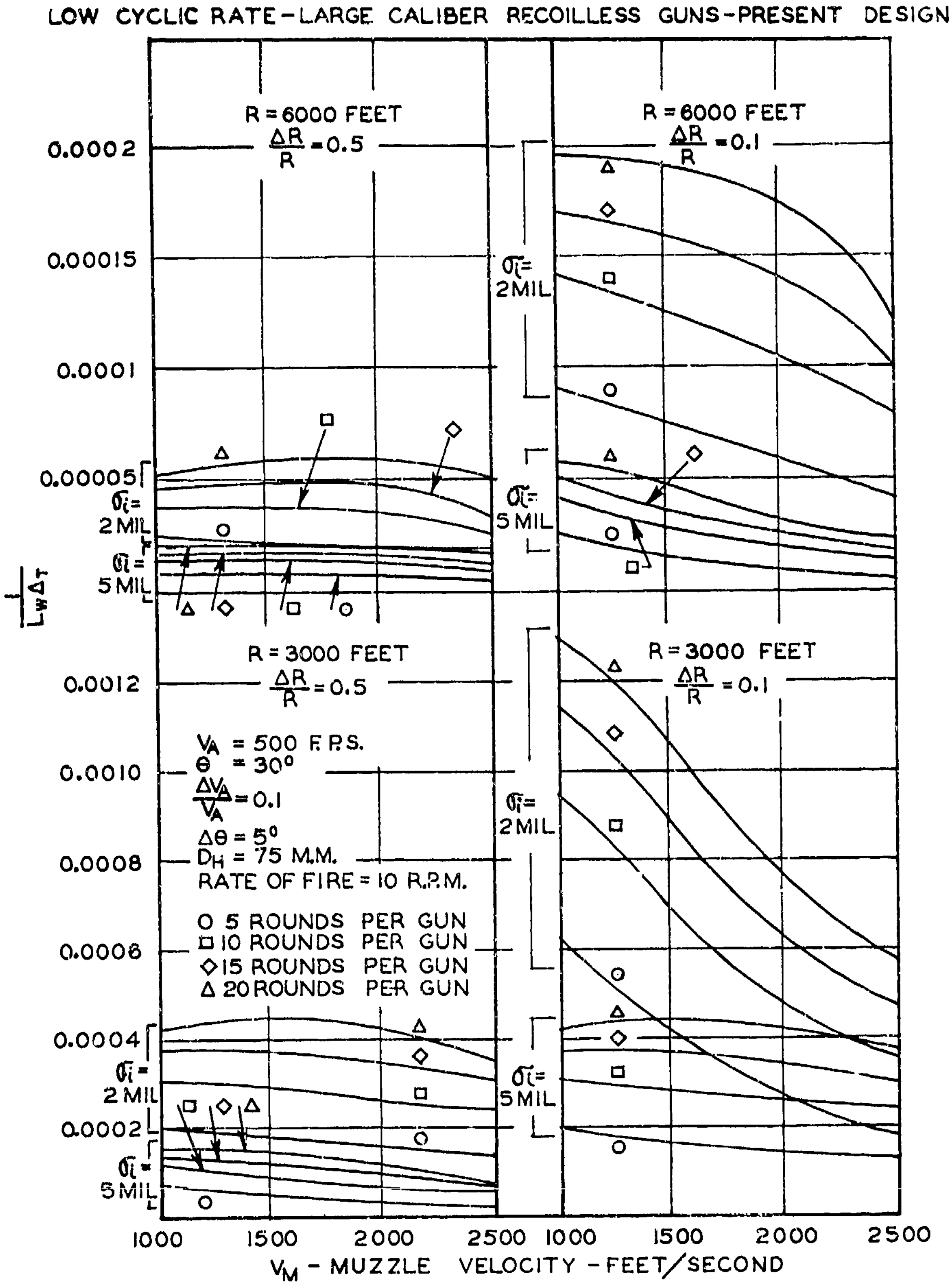


FIGURE 13A. RECIPROCAL OF AIRCRAFT ARMAMENT LOGISTIC FACTOR AS A FUNCTION OF GUN MUZZLE VELOCITY

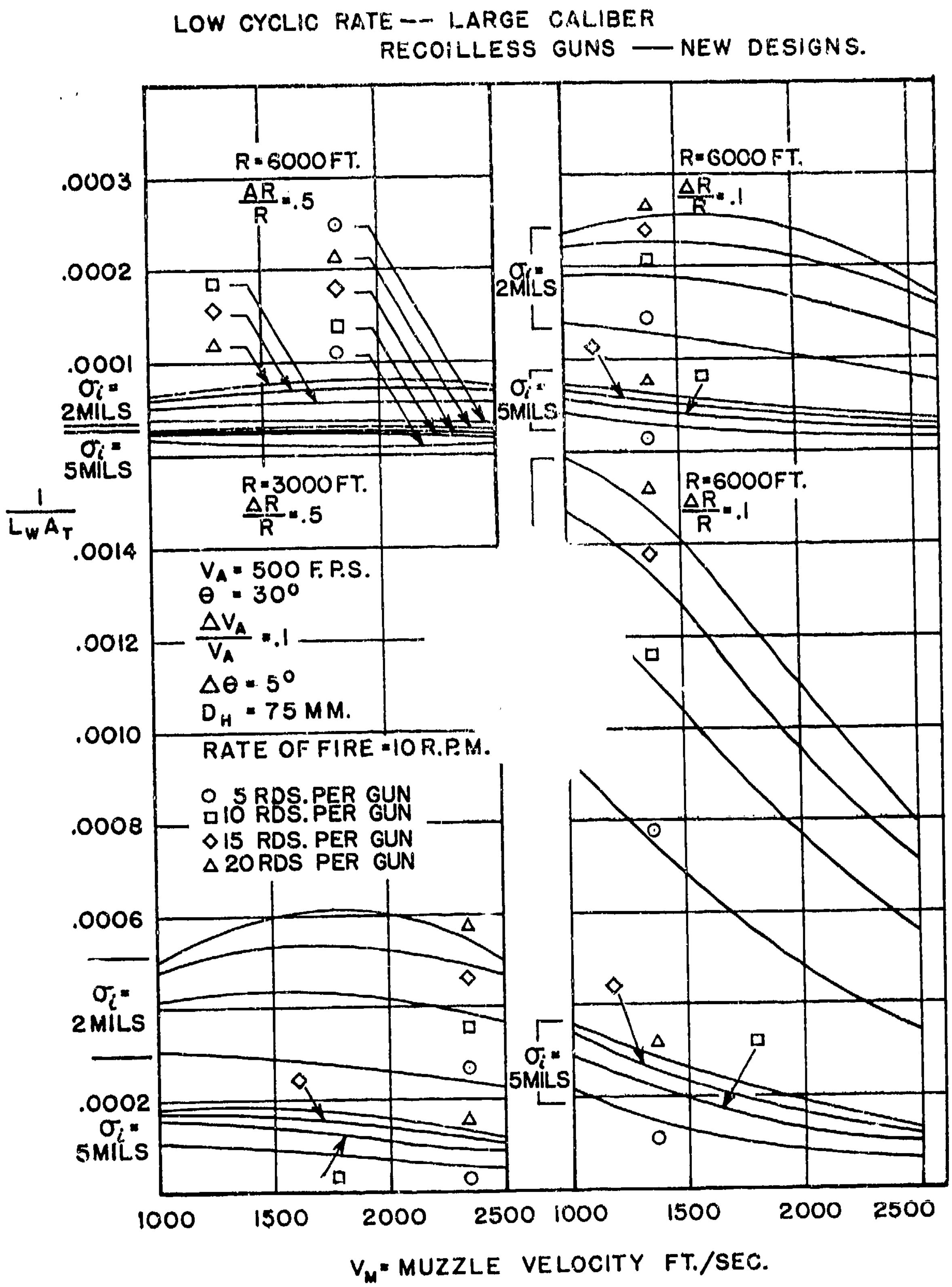


FIGURE 13 B. RECIPROCAL OF AIRCRAFT ARMAMENT LOGISTIC FACTOR AS A FUNCTION OF MUZZLE VELOCITY OF PROJECTILE FROM RECOILLESS GUN

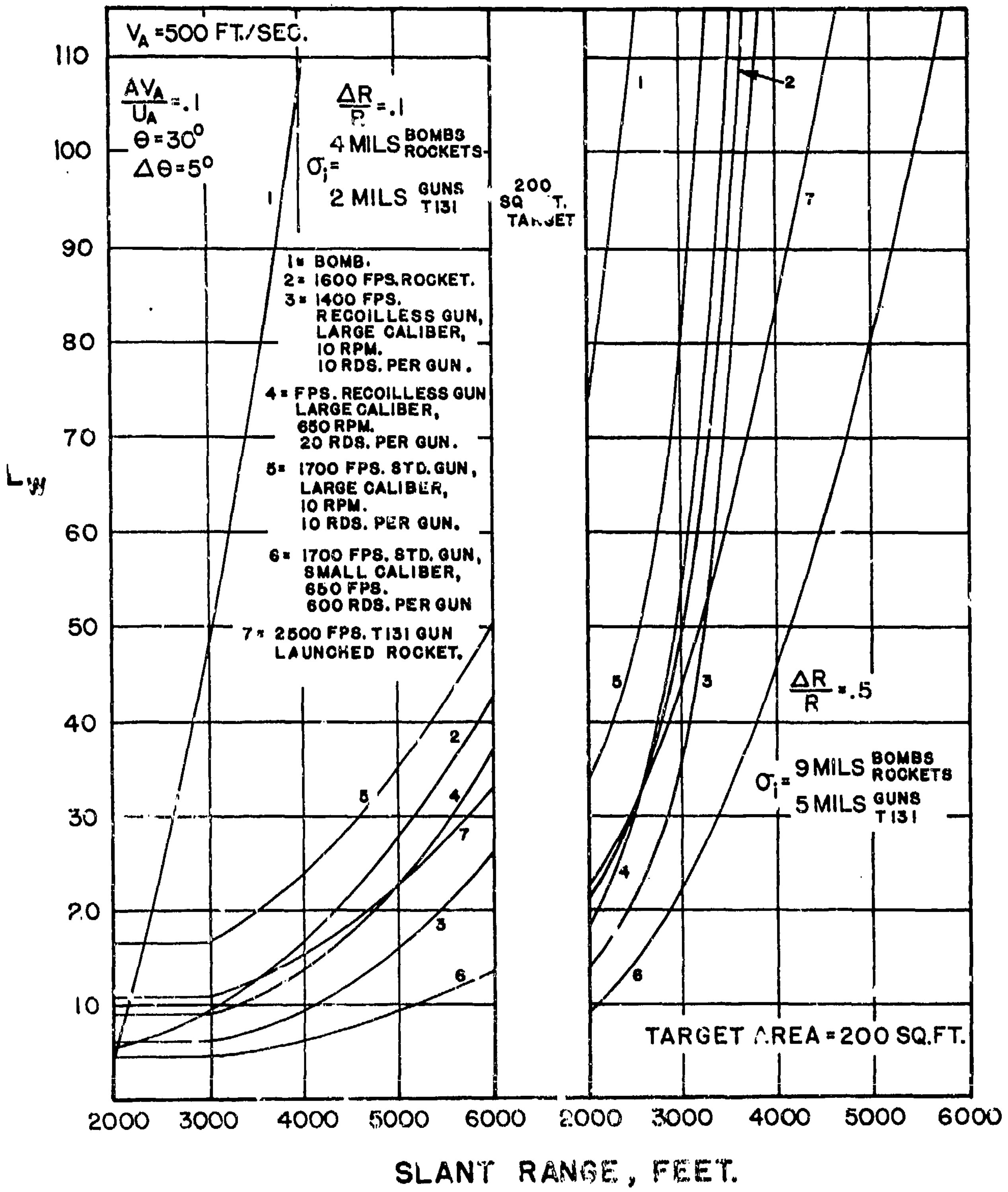


FIGURE 14A EFFECT OF SLANT RANGE UPON AIRPLANE ARMAMENT LOGISTIC FACTOR.

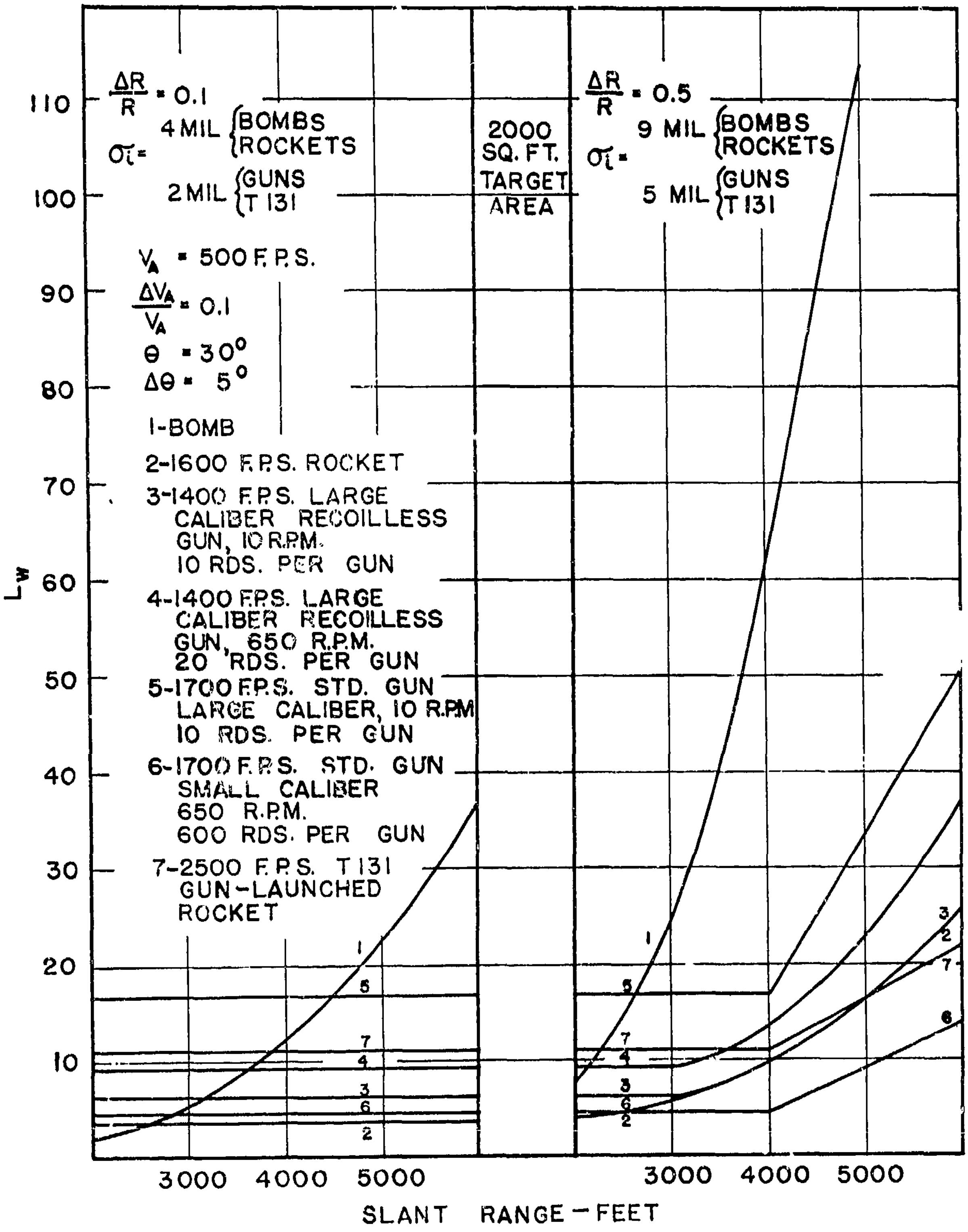


FIGURE 14 B. AIRPLANE ARMAMENT LOGISTIC FACTOR AS A FUNCTION OF RANGE TARGET AREA = 2000 SQUARE FEET

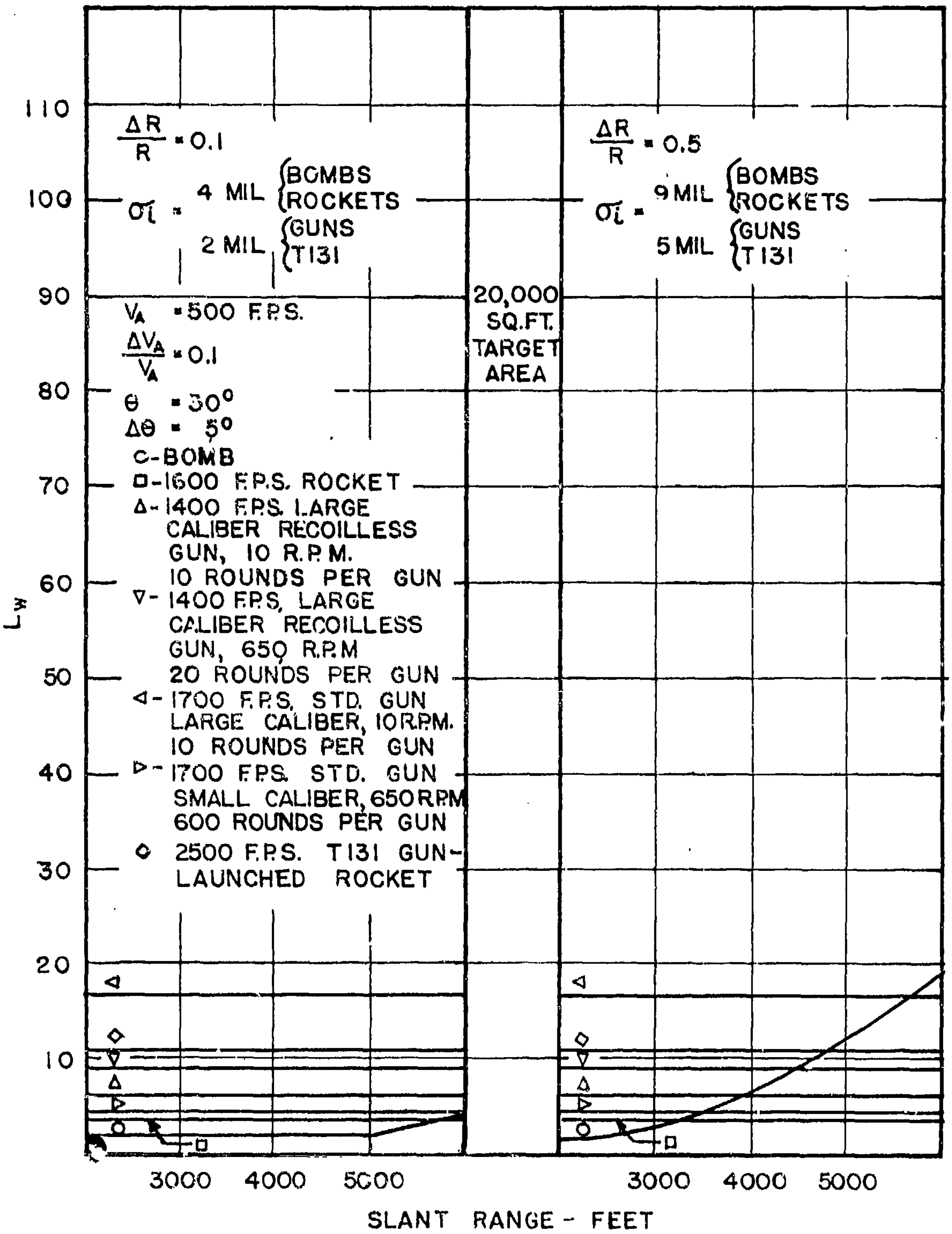


FIGURE 14C. AIRPLANE ARMAMENT LOGISTIC FACTOR AS A FUNCTION OF RANGE